Homework Assignments

Dynamical Systems I

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http://dynamics.mi.fu-berlin.de/lectures/

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Problem 37: Let Φ^t be a flow on a metric space X. Let x_0 be "stable", i.e.

$$\forall\,\varepsilon > 0 \quad \exists\,\delta > 0 \quad \forall\,x \in X \quad \Big(\ |x - x_0| < \delta \quad \Longrightarrow \quad \forall\,t \geq 0 \quad \big|\Phi^t(x) - \Phi^t(x_0)\big| < \varepsilon\ \Big)$$

Prove or disprove: x_0 is an equilibrium of the flow.

Problem 38: Consider the sequence

$$1,\ 1,\ 2,\ 3,\ 5,\ 8,\ 13,\ \ldots,$$

i.e.

$$x_n \;=\; \frac{1}{2^n\sqrt{5}}\left(\left(1+\sqrt{5}\right)^n - \left(1-\sqrt{5}\right)^n\right).$$

(i) Give an interpretation of this sequence via iterations of a suitable linear map $A : \mathbb{R}^2 \to \mathbb{R}^2$. Determine the linear map and prove your claim.

(ii) For which nonzero initial conditions $(\tilde{x}_1, \tilde{x}_2) \in \mathbb{Z}^2$ to the above iteration does the quotient

$$r_n \;=\; \tilde{x}_{n+1}/\tilde{x}_n$$

not converge to the "golden ratio" $g = \frac{1}{2}(1+\sqrt{5})$?

Problem 39: Let f be a differentiable vector field on $\mathbb{R}^3$. Show that a trajectory coincides with its ω-limit set, if the trajectory is an equilibrium or a periodic orbit.

8 extra points: Show the converse implication.

Problem 40: Let all eigenvalues of the linearization of an ODE at an equilibrium $x = 0$ possess strictly positive real part. Show that $x = 0$ is a repeller, preferably without using the Grobman-Hartman theorem.

Here we call $x = 0$ a repeller, if there exist $0 < \delta < \varepsilon$ such that for every initial condition x_0 with $0 < |x_0| < \delta$ there exists a positive time t with $|x(t)| = \varepsilon$.