Homework Assignments

Dynamical Systems I

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Problem 41: Consider the replicator equation

$$\dot{x}_i \;=\; x_i \left((Ax)_i - x^{\mathrm{T}} A x \right), \qquad i = 1, \ldots, n,$$

for the identity matrix A, and $x = (x_1, \ldots, x_N) \in \mathbb{R}^N$ with $x_i \geq 0$, $\sum_{i=1}^N x_i = 1$.

(i) Sketch the phase portraits for $N = 2, 3, 4$.

(ii) Describe the set of equilibria and the set of heteroclinic orbits for arbitrary N. In particular determine which equilibria are connected by heteroclinic orbits.

Hint: Enumerate equilibria x^* by subsets $M(x^*) = \{i;\; x_i^* \neq 0\} \subseteq \{1, \ldots, N\}$.

Problem 42: Let f be a differentiable vector field such that each trajectory is bounded. Prove or disprove: The ω-limit set depends continuously on the initial condition, i.e. $\lim |x_n - x| = 0$ for $n \to \infty$ implies $\lim \operatorname{dist}(\omega(x_n), \omega(x)) = 0$. Here $\operatorname{dist}(A, B) := \inf\{|a - b|;\; a \in A,\; b \in B\}$.

Problem 43: Show that the α-limit set of x_0 satisfies $\alpha(x_0) = \bigcap_{t \leq 0} \operatorname{clos} \gamma_-(x_0 \cdot t)$, where γ_- denotes the backwards orbit.

Problem 44: Whenever possible, give at least one example, each, of a global planar flow and an initial condition x_0 such that the ω-limit set $\omega(x_0)$ is

(i) empty;

(ii) unbounded;

(iii) disconnected;

(iv) not invariant.