Basic Questions of Dynamical Systems II

1. What is a Poincaré section to a periodic orbit of a flow?

2. What is a Poincaré map to a periodic orbit of a flow?

3. Formulate the Floquet theorem for a non-autonomous, time periodic, linear differential equation.

4. Formulate the Floquet theorem for an autonomous vector field, linearized at a periodic orbit.

5. What are Floquet multipliers and Floquet exponents of a periodic orbits of an autonomous vector field?

6. Why do periodic orbits of an autonomous vector field possess a trivial Floquet multiplier 1?

7. How is the rotation number of an (orientation preserving) homeomorphism $f : S^1 \to S^1$ defined?

8. How are existence and periods of periodic points related to the rotation number of a homeomorphism $f : S^1 \to S^1$?

9. Formulate the theorem of Denjoy for C^2-diffeomorphisms $f : S^1 \to S^1$.

10. How are local/global stable and unstable manifolds of a hyperbolic equilibrium of a vector field defined?

11. Formulate the theorem on the existence of local stable and unstable manifolds to a hyperbolic equilibrium of a vector field.

12. Are local stable and unstable manifolds to a hyperbolic equilibrium unique? What are the tangent spaces to stable and unstable manifolds at the equilibrium?

13. What is the (Bernoulli) shift on N symbols? Define the shift space, its topology, and the shift map.

14. How can we construct
    (a) periodic orbits of every period
    (b) a dense set of periodic orbits

(c) a dense orbit

for the shift on 2 symbols?

15. How does the shift on 2 symbols illustrate sensitive dependence on initial conditions?

16. What is the Smale horseshoe?

17. Formulate the theorem on the embedding of the shift into a C^0 horseshoe.

18. Formulate the theorem on the embedding of the shift into a C^1 horseshoe.

19. Sketch a horseshoe construction for the bouncing-ball map

$$\begin{aligned} \Phi_{k+1} &= \Phi_k + v_k, \\ v_{k+1} &= v_k - \gamma \cos(\Phi_k + v_k), \end{aligned}$$

under a suitable assumption on γ.

20. What is a hyperbolic structure?

21. What is a transverse homoclinic point of a diffeomorphism?

22. Formulate the λ-lemma.

23. How does a transverse homoclinic point give rise to shift dynamics? Sketch the relevant picture.

24. What is the Plykin attractor?

25. How is C^1 structural stability of a diffeomorphism defined?

26. Give at least two examples of structurally stable diffeomorphisms of the 2-torus.

27. What is a strange attractor? Sketch an example and list relevant properties.

28. How can the (global) center manifold be written as a fixed point of a contraction map on a suitable function space? Define the space and the contraction map.

29. How is the local center manifold to a non-hyperbolic equilibrium of a vector field defined?

30. Formulate the theorem on the existence of a local center manifold to a non-hyperbolic equilibrium of a vector field.

31. Under which assumptions on the vector field does a global center manifold to a non-hyperbolic equilibrium exist? Is the global center manifold unique?

32. Is the local center manifold to a non-hyperbolic equilibrium unique?

33. Let A be the linearization of a C^1 vector field on $\mathbb{R}^N$ at the equilibrium $x = 0$. Can the vector field possess periodic orbits arbitrarily near $x = 0$ if

    (a) the only purely imaginary eigenvalue of A is a simple eigenvalue zero.

    (b) the only purely imaginary eigenvalues of A are $\pm\mathrm{i}$, both simple.