Basic Questions of Dynamical Systems II

1. Formulate the ergodic theorem for the flow $\varphi^t : \mathbb{T}^2 \to \mathbb{T}^2$ to the differential equation

$$\dot{x} = \alpha, \ \dot{y} = \beta \ \text{ with } \ \beta/\alpha \notin \mathbb{Q}.$$

2. How is the Lie-derivative of a vector field f with respect to a vector field g defined, $f, g \in C^\infty(\mathbb{R}^N, \mathbb{R}^N)$. How is the Lie bracket of f and g defined? How is $\mathrm{ad}(A)$ for a matrix A defined?

3. Formulate the theorem about the normal form of a smooth vector field.

4. What is the normal form of a vector field f at an equilibrium, provided the adjoint representation $\mathrm{ad}_m(A^T)$ of the linearization has trivial kernel on the space of homogeneous polynomials of degree $m \geq 2$.

5. Formulate the theorem about the normal form of a smooth vector field using the adjoint representation of the transpose of its linearization.

6. Let the linearization A of a vector field f at an equilibrium be normal: $AA^T = A^T A$. Which additional symmetry does the $\mathrm{ad}(A^T)$ normal form of f to any finite order possess?

7. Let the linearization A of a vector field f at an equilibrium be diagonal. What are resonances? How do they relate to the $\mathrm{ad}(A^T)$ normal form of f?

8. What is a Hopf bifurcation? What is its normal form?