

10. Homework Assignment

**Dynamical Systems III**

Juliette Hell

<http://dynamics.mi.fu-berlin.de/lectures/>

**due date: Wednesday, 01.07.2015**

**Problem 1:** Consider the equation

$$\dot{x} = f(\mu, x), \quad \mu \in \mathbb{R}, \quad x \in \mathbb{R}^n, \quad n \geq 2.$$

Use the projection method for center manifold computation to express and prove conditions for

- (i) transcritical bifurcations at  $(\mu^0, x^0)$ .
- (ii) pitchfork bifurcations at  $(\mu^0, x^0)$ .

**Problem 2:** Consider this system of equations which models chemical reactions of two species

$$\begin{aligned}\dot{x} &= -\frac{a_1 x}{1 + b(\kappa - y)} + \frac{a_2(\kappa - x - y)}{1 + b(\kappa - x)}, \\ \dot{y} &= \frac{a_3(\kappa - x - y)}{1 + b(\kappa - y)} - \frac{a_4 y}{1 + b(\kappa - x)},\end{aligned}$$

with  $a_{1,2,3,4}, b, \kappa > 0$  and  $x, y, \kappa - x - y \geq 0$ .

- (i) Under which conditions on the parameters  $a_{1,2,3,4}, \kappa$  are there equilibria with  $x = y \neq 0$ .
- (ii) Under which conditions on the parameters does the linearization at the equilibria above have one strictly negative and one zero eigenvalue?
- (iii) Show, that there exists a generic cusp bifurcation with respect to the parameters  $a_1, a_4$ .