

11. Homework Assignment

Dynamical Systems III

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<http://dynamics.mi.fu-berlin.de/lectures/>

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Problem 1: Consider the Lorenz model

$$\begin{aligned}\dot{x} &= -y^2 - z^2 - ax + aF, \\ \dot{y} &= xy - bxz - y + G, \\ \dot{z} &= bxy + xz - z,\end{aligned}$$

with parameters a, b, F, G . Show, that there are equilibria at which a Fold-Hopf bifurcation takes place, that is, the linearization possesses a simple eigenvalue zero and a pair of purely imaginary eigenvalues.

Problem 2: Use the projection method on the center manifold in order to check that

$$\ddot{y} + \alpha \dot{y} + \beta y + y(1 - y) = 0, \quad y \in \mathbb{R}, \quad \alpha, \beta > 0,$$

undergoes a generic Hopf bifurcation at the origin under variation of α . Is the bifurcation sub- or supercritical? Determine the stability of the origin and of the periodic orbit in the different sectors of the bifurcation diagram.

Hint: Use the formula for the first Lyapunov coefficient from the Tutorium. You can also look it up e.g. in the Scholarpedia article on Hopf bifurcation.