12. Homework Assignment

Dynamical Systems III

Juliette Hell

http://dynamics.mi.fu-berlin.de/lectures/

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The exercises are for extra credit, i.e. points you obtain will count towards your "Übungsschein", however we will not use these questions to calculate the threshold necessary to obtain the "Übungsschein".

Problem 1: The truncated normal form for Fold-Hopf bifurcation reads

$$\begin{aligned}\dot{\xi} &= \beta_1 - \xi^2 + s\rho^2,\\ \dot{\rho} &= \rho(\beta_2 + \theta\xi + \xi^2),\\ \dot{\varphi} &= \omega_1 + \nu\xi,\end{aligned}$$

with $s = \pm 1, \theta \neq 0$. In class we derived the bifurcation diagram for $s = 1$, $\theta > 0$. Now study the bifurcation diagram for $s = -1$, $\theta < 0$. In other words, consider the equations

$$\begin{aligned}\dot{\xi} &= \beta_1 - \xi^2 + s\rho^2,\\ \dot{\rho} &= \rho(\beta_2 + \theta\xi + \xi^2).\end{aligned}$$

close to $\xi = 0, \rho = 0$.

(i) What is the bifurcation boundary in the (β_1, β_2)- plane?

(ii) What is the dynamics in each sector of the parameter plane?

(iii) Point out where a heteroclinic orbit connects two saddles with one-dimensional unstable manifolds.

Problem 2: Consider the normal form of the Bautin (or generalized Hopf) bifurcation given by

$$\dot{z} = (\beta_1 + i)z + \beta_2 z\,|z|^2 \pm z\,|z|^4\,, \qquad z \in \mathbb{C},\ (\beta_1, \beta_2) \in \mathbb{R}$$

(i) Derive an equation in polar coordinates, e.g. $z = \rho e^{i\varphi}$, in the case of $\pm = -$.

(ii) What is the bifurcation boundary? What is the dynamics in the different sectors of the parameter plane?

(iii) What happens of you choose $\pm = +$?