3. Homework Assignment

Dynamical Systems III

Juliette Hell

http://dynamics.mi.fu-berlin.de/lectures/

due date: Wednesday, 13.05.2015

Problem 1: Consider the second-order differential equation

$$\ddot{x} + \nu x + \mu \dot{x} + x^2 \dot{x} + x^3 = 0$$

with $\nu > 0$.

(i) Determine all equilibria of the system.

(ii) What is the linearization at the equilibria? Calculate the eigenvalues.

(iii) For which of the two parameters μ, ν does a Hopf bifurcation occur? Fix the other parameter and draw a bifurcation diagram.

Problem 2: Consider the system

$$\begin{aligned} \dot{x_1} &= \alpha \left[1 - x_1 x_2^2 + A(x_2 - 1)\right], \\ \dot{x_2} &= x_1 x_2^2 - x_2. \end{aligned}$$

where $A < 1$ and $\alpha \in \mathbb{R}$.

(i) Determine the equilibrium of the system.

(ii) Show, that there is a Hopf bifurcation in the parameter α.