4. Homework Assignment

Dynamical Systems III

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http://dynamics.mi.fu-berlin.de/lectures/

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Problem 1: Consider the differential equation

$$\dot{x} = -x\left(\mu^2 + x^2\right), \qquad x \in \mathbb{R},\ \mu \in \mathbb{R}.$$

(i) Determine the equilibria and their stability in dependence of μ. Which value of μ could yield a bifurcation at the origin? What happens?

(ii) Solve the equation explicitly by hands. Describe the interval of existence and the asymptotics of the maximal solution in dependence of the initial conditions.

Problem 2: Let $A(\alpha)$, $\alpha \in \mathbb{R}$ denote a smooth family of $n \times n$ matrices of the form

$$A(\alpha) := \left(\begin{array}{cc|c} \lambda(\alpha) & -\omega(\alpha) & 0 \\ \omega(\alpha) & \lambda(\alpha) & 0 \\ \hline 0 & 0 & B(\alpha) \end{array}\right)$$

Assume, that $\lambda(\alpha) \pm i\omega(\alpha)$ are simple eigenvalues. Let $p(\alpha)$ be the left eigenvector of $A(\alpha)$ with eigenvalue $\lambda(\alpha) - i\omega(\alpha)$, i.e.

$$A^t(\alpha)p(\alpha) = \left[\lambda(\alpha) - i\omega(\alpha)\right] p(\alpha).$$

Furthermore denote by $q(\alpha)$ the right eigenvector of $A(\alpha)$ with eigenvalue $\lambda(\alpha) + i\omega(\alpha)$. You may assume, that $p(\alpha), q(\alpha)$ are smooth with respect to α.

(i) Check, that $\langle p(\alpha), q(\alpha)\rangle \neq 0$, hence w.l.o.g. $\langle p(\alpha), q(\alpha)\rangle = 1$.

(ii) Prove, that $\lambda'(0) = \Re\left(\langle p(0), A'(0)p(0)\rangle\right)$.

Hint: Differentiate a suitable scalar product with respect to α.
Remark: The inner product of $\mathbb{C}^n$ is defined by

$$\langle p, q\rangle := \sum_{i=1}^{n} p_i \bar{q}_i.$$