6. Homework Assignment

Dynamical Systems III

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Problem 1: Find the bifurcation points of the following differential equations with parameter $\lambda \in \mathbb{R}$. Determine the type of bifurcation and draw the bifurcation diagram close to the bifurcation.

(i) $\dot{x} = \lambda x - \log(1 + x)$.

(ii) $\dot{x} = -x + \lambda \tanh x$.

(iii) $\dot{x} = \lambda + x - \log(1 + x)$.

Problem 2: Consider the differential equation

$$\dot{x} = \lambda \log x + x - 1.$$

(i) Find the bifurcation point.

(ii) Determine the type of bifurcation and draw the bifurcation diagram.

(iii) Find a coordinate change $(x, \lambda) \mapsto (y, \mu)$ under which the equation for y has the form

$$\dot{y} = P_k(\mu, y) + O(y^{k+1}),$$

where P_k contains all terms of the normal form for the corresponding bifurcation up to order k.