

7. Homework Assignment

**Dynamical Systems III**

Juliette Hell

<http://dynamics.mi.fu-berlin.de/lectures/>

**due date: Wednesday, 10.06.2015**

**Problem 1:** Consider the equation

$$\dot{x} = x^2 + b^2 - a^2, \quad x \in \mathbb{R}, \quad (a, b) \in \mathbb{R}^2.$$

- (i) Determine the equilibria of the system.
- (ii) Compute the bifurcation candidates, i.e. solve

$$\begin{aligned} f(a, b, x) &= 0, \\ \frac{\partial f}{\partial x}(a, b, x) &= 0, \end{aligned}$$

and draw the solutions in the parameter plane.

- (iii) Let  $(a(s), b(s))$ ,  $s \in \mathbb{R}$  be a smooth curve in the parameter plane with  $\nu(s) = (a'(s), b'(s))$ ,  $\|\nu\| = 1$ . When does the curve yield a bifurcation? Determine the bifurcation type. What happens in cases in which no bifurcation occurs? Sketch the different possibilities in the parameter plane.

**Problem 2:** Consider the equation

$$\dot{x} = f(x, \mu), \quad f \in C^\infty(I \times \mathbb{R}, \mathbb{R}), \quad I = [-1, 1], \quad \mu \in \mathbb{R}.$$

- (i) Let  $k \in \mathbb{N}$ . Give an example of  $f$  such that there is at most one equilibrium for  $\mu \leq 0$  and  $k$  equilibria for  $\mu > 0$ .

*Hint:* Assume, that  $f$  is a polynomial in  $x$ .

- (ii) Let  $k = \infty$ . Give an example of  $f$  such there is at most one equilibrium for  $\mu \leq 0$  and infinitely many equilibria for  $\mu > 0$ .

*Hint:* Define

$$g(x, \mu) := \exp\left(\frac{-1}{|x^2 - \mu|}\right)$$

Consider now

$$f(x, \mu) := \left[1 + e^\mu \left(1 - \frac{x^2 - \mu}{2}\right) \sin\left(\frac{1}{x^2 - \mu}\right)\right] g(x, \mu).$$

- (iii) Can there be function  $f$  such that there is most one equilibrium for  $\mu \leq 0$  and a whole interval of equilibria for  $\mu > 0$ ?

*Hint:* Consider an exponentially flat function as in (ii).