

8. Homework Assignment

Dynamical Systems III

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<http://dynamics.mi.fu-berlin.de/lectures/>

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Problem 1: Consider the equation

$$\dot{x} = f(\mu, x), \quad \mu \in \mathbb{R}^2, \quad x \in \mathbb{R}^2.$$

Assume, that there exists an equilibrium (μ^0, x^0) where a generic Hopf bifurcation takes place with respect to μ_1 . Prove, that the system of equations

$$\begin{aligned} f(\mu, x) &= 0, \\ \text{tr}\left(\frac{\partial f}{\partial x}(\mu, x)\right) &= 0, \end{aligned}$$

can be solved locally and has a curve of solutions through (μ^0, x^0) .

Problem 2: Give an example of a polynomial Hamiltonian system $H(\mu, x, y)$, $\mu \in \mathbb{R}, (x, y) \in \mathbb{R}^2$ with parameter μ exhibiting a homoclinic orbit at the origin.