9. Homework Assignment

Dynamical Systems III

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http://dynamics.mi.fu-berlin.de/lectures/

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Problem 1: Consider the equation

$$\dot{x} = x\left((x-\mu_1)^2 - \mu_2\right), \qquad x, \mu_1, \mu_2 \in \mathbb{R}.$$

(i) Do generic codim-2 bifurcations occur?

(ii) In the parameter plane, determine the curves along which codim 1 bifurcations happen. Indicate in each sector of this diagram the number of equilibria and their stability, and draw a representative phase portrait.

Hint: Use colors!

Problem 2: Consider the ordinary differential equation

$$\dot{x} = \alpha_1 + \alpha_2 x - x^3 + F(\alpha, x),$$

with $F(\alpha, x) = O(x^4)$.

(i) Show the local existence of a curve Γ of saddle-node bifurcations through $(0,0)$. *Hint:* Parametrize Γ by x, i. e., $\Gamma = (\alpha_1^*(x), \alpha_2^*(x), x)$.

(ii) Compute the leading order terms of the Taylor expansion of α_1^*, α_2^* at $x = 0$ and determine the projection of Γ in the parameter plane, i. e. the bifurcation boundary.

(iii) How many equilibria are present in the different sectors of the parameter plane enclosed by the bifurcation boundary. Draw and describe the corresponding phase portraits? Justify your claims.