1.) Define a strongly continous linear Semigroup and its infinitesimal generator. Give two examples.

2.) Formulate and prove the thm about the growth cond of a sc sgr

3.) What are the definitions of
(1) a densely defined operator
(2) a closed operator
(3) the resolvent set
(4) and the spectrum of a linear operator?

4.) How are strongly continous semigroups and infinitesimal generator related?

5.) Define weak derivative. Are they unique? Prove!

6.) Formulate the Lax-Milgram Thm

7.) Let A be a linear operator. Define its prop: closable, dissipative and maximal dissipative and the extension of A.

8.) Under which cond. does resolvent estimate

$$\|(A-\lambda)^n\| \le \frac{1}{(\mathrm{Re}\,\lambda)^n} \quad \forall\, \mathrm{Re}\,\lambda > 0,\ n = 1, 2, 3 \ldots$$

hold? can you conclude further results?

9) State the resolvent identity and prove that resolvents of dd, cl operator A are analytic.

Prove or disprove that $A(id - 1/\lambda\, A)^{-1}$ is bd. operator.

10) Define strong and mild sol of (1) $\begin{cases} u'(t) = Au(t) + f(t), & t \geq 0 \\ u(0) = u_0 \end{cases}$

11) Give sufficient cond, for a mild sol. to be unique

12) Formulate the Gronwall Lemma.

13) Are mild sol of semilinear eq $\begin{cases} u'(t) = Au(t) + f(u(t)) \\ u(0) = u_0 \end{cases}$ unique? (local solution)
When do they exist?

14) " — strong — "

15) Solve the linear wave eq with sgr theory.

16) Define analytic sgr.

17) Define the sectorial property of an operator A.

18) Define

19) Let $A \in \mathcal{H}(M, \beta, \kappa)$ and pick any $0 < \kappa' < \kappa$, $\beta' > \beta$.
What can you conclude about A?

20) Which assumptions are sufficient to conclude that a mild sol of an analytic sgr is unique? ((local solution))

21) Give three different definitions of fractional powers of an operator and define its range. Is it a Banach space?

22) Name (4-6) prop of fractional powers of $A \in \mathcal{H}(M, \beta, \kappa)$

23.) Let A be the inf gen of s.c sgr $(T(t))$ and $\lambda \in \rho(A)$ resolvent set. Prove that $(\lambda - A)^{-1}$ is bounded.

24.) Consider the operator $(Au)(x) = u''(x) = \frac{d^2}{dx^2} u(x)$, consider $X = L^2(0,\pi)$ and periodic bc.
give its domain and show its dissipativity and define dissipative operator.

25) Consider $Au = u'$ and $X = L^2(\mathbb{R})$ with $D(A) = C_c^1(\mathbb{R})$
Show that A and $-A$ are dissipative

26) Prove that any strong sol is a mild sol and hence unique.

27) Let $A: \mathbb{R}^N \to \mathbb{R}^N$ be linear. Define

$$T(t)u := \exp(tA)u = \sum_{k=0}^{\infty} \frac{(tA)^k}{k!}$$

Show that $T(t)$ defines a strongly continous semigroup. What is its infinitesimal generator?

28) Let X be Banach space and $T(t), t \geq 0$ a strongly continous semigroup with infinitesimal generator A. Suppose that λ is an eigenvalue of A with corresponding eigen fct u_λ.

Show that

$$T(t)u_\lambda = e^{\lambda t} u_\lambda$$

holds $\forall t \geq 0$

29) State a thm to solve first order PDE's with characteristics

30) Formulate the Sobolev embedding thm.

31) Prove the existence of a sol of the heat equation $u_t = -\Delta u + f(t)$, $u(0) = u_0$ with Dirichlet boundary condition

32) How can you use the Yoshida regularization to solve $\dot{u}(t) = Au(t)$, $u(0) = u_0$.

33) Determine the β, M values of the growth cond of the sgr $T(t) = e^{At}$ with matrix $A \in \mathbb{R}^{d \times d}$