Infinite-Dimensional Dynamics

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Assignment 4
due date: Monday, November 16, 2015, 5 p.m.

Problem 13: Let $u \in W^{1,p}([0,1])$, $1 \leq p < \infty$, i.e. $u, v \in L^p$ such that for all $\varphi \in C_0^\infty$

$$\int_0^1 u\varphi' = -\int_0^1 v\varphi.$$

With clever choices of φ, prove that $u \in C^{0,\gamma}$ for all $\gamma \leq 1 - 1/p$. (Assume u to be continuous, if necessary.)

Problem 14: Let B be the unit ball in $\mathbb{R}^n$. Consider $u_\beta : B \to \mathbb{R}$, $x \mapsto u_\beta(x) = |x|^\beta$. Find all real values of β such that

(i) $u_\beta \in C^{0,\alpha}(B)$;

(ii) $u_\beta \in W^{1,p}(B)$.

Problem 15: Let $\Omega \subset \mathbb{R}^n$ be open and connected. Consider $u \in W^{1,p}(\Omega)$, $1 \leq p < \infty$, with vanishing weak derivative

$$\int_\Omega u\partial_i\eta = 0 \qquad \forall \eta \in C_0^\infty(\Omega),\ i = 1, \dots, n.$$

Prove that u is a constant function.

Problem 16: Again, let $\Omega \subset \mathbb{R}^n$ be open and connected, withsmooth boundary. Furthermore, let $k \in \mathbb{N}_0$, and $0 < \alpha < \beta \leq 1$. Prove or disprove that the following embeddings exist:

(i) $C^{k,1}(\bar{\Omega}) \hookrightarrow C^{k+1}(\bar{\Omega})$,

(ii) $C^{k+1}(\bar{\Omega}) \hookrightarrow C^{k}(\bar{\Omega})$,

(iii) $C^{k,\alpha}(\bar{\Omega}) \hookrightarrow C^{k,\beta}(\bar{\Omega})$,

(iv) $C^{k,\alpha}(\bar{\Omega}) \hookrightarrow C^{k}(\bar{\Omega})$. Which of the existing embeddings is compact?