Infinite-Dimensional Dynamics

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Assignment 9
due date: Monday, January 18, 2016, 5 p.m.

Problem 33: Let $u^n(\cdot)$ be mild solutions of

$$u_t^n(t) = Au^n(t) + f^n(t)$$

with initial values $u^n(0) = u_0^n$. Assume the existence of the limits

$$\lim_{n\to\infty} u_0^n \;=\; u_0 \quad \text{in } X; \qquad \lim_{n\to\infty} f^n \;=\; f \quad \text{in } C^0([0,\tau],X).$$

Do the solutions u^n then converge to a limit solution u in $C^0([0,\tau],X)$? What can you say about $t \in [0,\infty)$?

Problem 34: Consider the equation

$$u_t(t) = Au(t) + f(u(t))$$

with $A \in G(M,\beta)$ on the Banach space X. Let f be locally Lipschitz with the linear growth condition

$$\|f(u)\|_X \;\leq\; C(1 + \|u\|_X)$$

Prove the existence of a global mild solution $u(t)$, $t \in [0,\infty)$. (In particular, there exists a global mild solution for nonlinearities f that are *globally* Lipschitz.) Find a condition on f that provides global *strong* solutions.

Problem 35: Consider the equation

$$u_t(t) = Au(t) + f(u(t))$$

and the shift operator

$$(S_\tau u)(t) = u(t+\tau)$$

Prove: If u is a mild solution on $t \in [0,T_1]$ and $S_{T_1}u$ is a mild solution on $t \in [0,T_2]$, then u is a mild solution on $t \in [0,T_1+T_2]$, i.e. mild solutions can be concatenated.

Problem 36: Consider the delay differential equation

$$\dot{u}(t) = au(t) + bu(t-1) + f(t) \tag{1}$$

where $a,b \in \mathbb{R}$, and given prehistory $u_0(\vartheta)$, $-1 \leq \vartheta \leq 0$, $u_0 \in C^0$. Suppose $f \in C^\infty$.

(i) Use the variation-of-constants formula to find a mild solution for equation (1).

(ii) Is the solution unique?

(iii) Is the mild solution strong?