Homework assignment

Infinite-Dimensional Dynamical Systems

Bernold Fiedler, Hannes Stuke
http://dynamics.mi.fu-berlin.de/lectures/

Due Tuesday, May 03, 2016, 10:15

Let X be a Banach space and $T(t)$ a compact, strongly continuous semigroup on X.

Problem 1: Consider the following equation

$$\dot{x} = x(1 - x^2), \qquad x \in \mathbb{R}.$$

Describe all possible ω - limit sets $\omega(K)$ of compact intervals K. Which of these sets, if any, attract all compact sets?

Problem 2: $T(t)$ is called point/compact/bounded dissipative if there exists a bounded set B, which attracts all points/compact sets/bounded sets. Which of the following three properties implies which other (prove/disprove):

(i) point dissipative;

(ii) compact dissipative;

(iii) bounded dissipative?

Problem 3: Prove or disprove the equivalence of the following two definitions of invariance for compact sets I:

(i) I is invariant, if $T(t)I = I$ for all $t \geq 0$.

(ii) I is invariant, if for all $x_0 \in I$ there exists a prehistory $\varphi(t)$ of x_0, for $t \leq 0$, and $T(t)x_0 \in I$ holds for all $t \geq 0$.

Problem 4: Assume, that $T(t)$ has a compact invariant set I, that is $T(t)I = I$ for all $t \geq 0$. Assume furthermore, that every prehistory in I is unique. Show, that the semigroup $T(t)$ restricted to I can be extended to a strongly continuous group, i.e. the semigroup axioms (including strong continuity) hold for all $t \in \mathbb{R}$.