Homework assignment

Infinite-Dimensional Dynamical Systems

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Problem 5: Consider the flow ϕ of the pendulum equation

$$\ddot{x} + \nu\dot{x} + \sin x = 0, \qquad \nu \in \mathbb{R},$$

for the two cases $x \in S^1 := \mathbb{R}/2\pi$ and $x \in \mathbb{R}$.

(i) For which real ν is ϕ dissipative?

(ii) Depending on ν, what is the maximal compact and invariant set of ϕ ?

Problem 6: Does there exist a strongly continuous semigroup $T(t)$ with nonequilibrium solutions such that all solutions are stationary after time $t = 1$, that is $T(t) = T(1)$ for all $t \geq 1$? Does your example possess a Lyapunov function?

Problem 7: Let $T(t)$ be a compact dissipative semigroup. Show that the α - limit set $\alpha(x_0)$ of any x_0, with respect to a bounded prehistory $\varphi(t)$ of x_0, is nonempty, compact, invariant, and connected.

Problem 8: For fixed $0 < \beta \leq \frac{1}{3}$, construct the closed Cantor set C starting from the closed unit interval: at stage $n \geq 1$, a centered open interval is removed, from each remaining interval, to generate two new remaining closed intervals of length β^n from it. The Cantor set is the limit of this process.
Determine the topological dimension, the Hausdorff dimension, and the capacity of C. What happens for $\beta > \frac{1}{3}$?