Homework assignment

Infinite-Dimensional Dynamical Systems

Bernold Fiedler, Hannes Stuke

http://dynamics.mi.fu-berlin.de/lectures/

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Problem 13: Let $T(t,\alpha)$, $0 \le \alpha < \alpha_0$ be a family of parameter dependent semigroups on X. Assume, that $T(t,\alpha)$ is locally uniformly continuous in α, that is for any fixed $t > 0$ and any bounded subset $M \subset X$

$$\lim_{\alpha\to 0} \sup_{x\in M} \|T(t,\alpha)x - T(t,0)x\| = 0.$$

Furthermore assume that all $T(t,\alpha)$ are point dissipative with the same attracting set B. Prove or refute that the global attractors $\mathcal{A}_\alpha$ of the semigroups satisfy

(i) $\lim_{\alpha\to 0} \operatorname{dist}(\mathcal{A}_\alpha, \mathcal{A}_0) = 0$,

(ii) $\lim_{\alpha\to 0} \operatorname{dist}(\mathcal{A}_0, \mathcal{A}_\alpha) = 0$.

Note: $\operatorname{dist}(M, \tilde{M}) := \sup_{x\in M} \inf_{y\in \tilde{M}} \|x - y\|$ is asymmetric.

Problem 14: Consider the map $F : c_0 \to c_0$,

$$(F(x))_n = x_n^2.$$

Prove, that $DF(x)$ is compact, but F is not compact.

Definition: The space c_0 is defined as follows

$$c_0 := \left\{ (x_n)_{n\in\mathbb{N}} : x_n \in \mathbb{R},\ \lim_{n\to\infty} x_n = 0 \right\},$$

with norm $\|\cdot\|_\infty$.

Problem 15: Consider a dissipative C^2 - flow on $\mathbb{R}^N$ with differential equation

$$\dot{x} = f(x), \qquad x(0) = x_0 \in \mathbb{R}^N.$$

and global attractor $\mathcal{A}$. Assume that,

$$\operatorname{rank} Df(\tilde{x}) \le k \le N, \qquad \forall \tilde{x} \in \mathcal{A}.$$

Show that the Hausdorff dimension of $\mathcal{A}$ satisfies $\dim_H \mathcal{A} \le k$.

Problem 16: Consider the Levin-Nohel nuclear reactor equation

$$\dot{x}(t) = -\int_{-1}^{0} a(s)g(x(t+s))ds, \tag{1}$$

for $x \in C^0([-1,0])$. Let $a(s)$ be positive, continuous and g continuously differentiable with $g(x) \geq -C_g$ and $xg(x) > 0$ for $x \neq 0$. Show that the semiflow of (1) is compact, dissipative and the global attractor has finite box-counting dimension.