Homework assignment

Infinite-Dimensional Dynamical Systems

Bernold Fiedler, Hannes Stuke

http://dynamics.mi.fu-berlin.de/lectures/

Due Tuesday, May 31, 2016, 10:15

Problem 17: Consider smooth solutions $u = u(t, x)$ of the semilinear heat equation $u_t = u_{xx} + f(x, u)$ on the unit interval $x \in \Omega := (0, 1)$ with Dirichlet boundary conditions. Let $F_u = f$. Show, that the functional

$$V(u) := \int_0^1 \left(\frac{1}{2}|u_x|^2 - F(x, u) \right) dx$$

is a Lyapunov function.

Problem 18: For $\varphi \in X := C^0 \left([-1, 0], \mathbb{R}^N \right)$ consider the delay equation

$$\dot{x}(t) = f(x_t), \ x_0 = \varphi \tag{1}$$

with notation $x_t(\vartheta) := x(t + \vartheta)$, $-1 \leq \vartheta \leq 0$. Assume $f : X \to \mathbb{R}^N$ is locally Lipschitz and satisfies $|f(\varphi)| \leq C(1 + \|\varphi\|_\infty)$ for some constant $C > 0$ and all $\varphi \in X$. Show that solutions of (1) exist globally.

Problem 19: Calculate the topological, box-counting (fractal, capacity) and Hausdorff dimension of

(i) $M := \left\{ \frac{1}{n}, \ n \in \mathbb{N} \right\}$,

(ii) the following Cantor set C_α with parameter $0 < \alpha < 1$: Start with the unit interval. From each remaining interval of length L remove a centered, open interval of length αL to obtain two new intervals. Continue recursively. The remaining set of this countable process is called C_α.

Problem 20: Consider the linear delay equation

$$\dot{x}(t) = -x(t - r),$$

on $C^0 \left([-r, 0], \mathbb{R} \right)$. Prove or disprove, that the semiflow of the equation is dissipative for all $r > 0$.