Homework assignment

Infinite-Dimensional Dynamical Systems

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Problem 21: Rewrite the ODEs for the chaotic Lorenz attractor as a regulatory network. Find a minimal feedback vertex set. Is this a set of determining nodes? Interpret your answer for chaotic trajectories on the Lorenz attractor. (You may assume the Lorenz ODEs to be dissipative). Can you find a vertex which does not define a feedback vertex set, but is still determining?

Problem 22: Consider the ordinary differential equation

$$\dot{x} = -x^3, \qquad x \in \mathbb{R}.$$

The global attractor is the trivial equilibrium $x = 0$. Consider the forward Euler discretization of the flow, i.e.

$$x_{n+1} = x_n - \epsilon x_n^3$$

for a fixed $\epsilon > 0$. Describe the behaviour of the numerical solutions. Is the ODE attractor approximated by the numerics for $\epsilon \to 0$?

Problem 23: Choose a hyperbolic matrix $A \in \mathrm{SL}_2(\mathbb{Z})$ and let φ be smooth on the 2-torus $\mathcal{T}^2 = \mathbb{R}^2/\mathbb{Z}^2$ with values in $\mathbb{R}$. Compare our two upper estimates of the capacity with the Lyapunov dimension for the Mallet-Paret & Yorke global attractor of the skew product $x_{n+1} = Ax_n$, $y_{n+1} = \lambda y_n + \varphi(x_n)$ on $(x, y) \in \mathcal{T}^2 \times \mathbb{R}$. How do the Lyapunov dimension and your estimate depend on scalings $\tilde{y} = \sigma y$ of y?

Problem 24: Consider the setting of Problem 23 with $A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$. Show that the graph Γ of the map $\Phi : \mathcal{T}^2 \to \mathbb{R}$ given by

$$y = \Phi(x) := \sum_{k=1}^{\infty} \lambda^{k-1} \varphi\left(A^{-k} x\right)$$

satisfies the following properties:

(i) Φ is C^k for $0 < \lambda < ((3 - \sqrt{5})/2)^k$, $k = 0, 1, 2, \ldots$;

(ii) Γ is the only graph of any map $\Phi : \mathcal{T}^2 \to \mathbb{R}$ such that Γ is invariant under the given iteration;

(iii) Γ coincides with the global attractor, for any $0 < \lambda < 1$.