Homework assignment

Infinite-Dimensional Dynamical Systems

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http://dynamics.mi.fu-berlin.de/lectures/

Due Tuesday, June 14, 2016, 10:15

Problem 25: Let $M \subset X := C([-1,0],\mathbb{R})$ be a finite-dimensional C^2 - submanifold. Assume that M is positively invariant under the local semiflow $T(t)$ to $\dot{x}(t) = f(x_t)$. Here $x_t(\vartheta) := x(t+\vartheta)$, for $-1 \leq \vartheta \leq 0$. Assume that the vector field f induces a locally Lipschitz vector field on M. Show, that without any other knowledge of f itself, f is already determined by M.
In other words, show, that $M \subset C^1([-1,0],\mathbb{R})$ and that $\dot{x}_t$ is already determined by $x_t \in M$ alone. Then conclude the claim.

Problem 26: Let A be a self-adjoint operator (look up the definition!) with dense domain D in a Hilbert space X . Let $\|\cdot\|$ denote the scalar product norm on X. Show that the set of $x \in D$ with $\|Ax\| \leq 1$ is closed and convex in X.

Problem 27: Consider N-dimensional smooth flows which depend smoothly on a parameter α. Assume a fixed attracting ball, for all α, so that the global attractors $\mathcal{A}_\alpha$ exist.

Prove or disprove that the dependence of the "dimension" of $\mathcal{A}_\alpha$ on α is

(i) upper semicontinuous;

(ii) lower semicontinuous.

Problem 28: In the setting of problem 27, prove or disprove that the dependence of $\mathcal{A}_\alpha$, as a set, on α is

(i) upper semicontinuous;

(ii) lower semicontinuous.

Here upper semicontinuity means that any open neighborhood of A_0 contains A_α, for sufficiently small $|\alpha|$. Lower semicontinuity means that the ε-neighborhood of A_α contains A_0, for sufficiently small $|\alpha|$.