Homework assignment

Infinite-Dimensional Dynamical Systems

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Due Tuesday, June 21, 2016, 10:15

The inertial manifolds of problems 29 – 31 are considered in our finite-dimensional ODE setting.

Problem 29: Interpret the manifold constructed in problem 24 as an inertial manifold. Discuss the assumptions of our inertial manifold theorem from the lecture in that example. Which aspects and cases does our theorem miss?

Problem 30: Is the inertial manifold constructed in the lecture also negatively invariant?

Problem 31: Let M be an inertial manifold. Is it unique? Is it unique under the condition that the dimension is minimal?

Problem 32: Consider solutions $u = u(t, x)$ of the linear heat equation

$$u_t = u_{xx}, \qquad x \in I := (0, 1)\,, \tag{1}$$

with Dirichlet boundary conditions and initial condition $u(0, x) = u_0(x)$. We define a semiflow $T(t)$ on the unit sphere $X := \{\tilde{u} \in L^2(I)\,, \|\tilde{u}\|_{L^2} = 1\}$ by solutions to equation (1),

$$T(t)u_0 := u(t, \cdot)/\|u(t, \cdot)\|_{L^2}.$$

(i) Prove that $T(t)$ is a globally defined semiflow for $t \geq 0$.

(ii) Describe all possible ω - limit sets. Is $T(t)$ compact?

(iii) Describe all equilibria and their heteroclinic orbits.

(iv) Can there exist solutions $T(t)u_0$, $t \leq 0$, with an α - limit set which is not an equilibrium?
Free extra: What is the α - limit set in your example?