Homework assignment

Infinite-Dimensional Dynamical Systems

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Due Tuesday, June 28, 2016, 10:15

Problem 33: Let $T(t)$ be a dissipative semiflow on a Banach space X, with $T(t) \in C^1$ for all $t > 0$. Assume the existence of a connected inertial C^1 - manifold M.
Let $\mathcal{E}$ denote the set of equilibria. Assume that all equilibria are hyperbolic without negative real eigenvalues. Denote by n_x the number of positive, real eigenvalues of equilibrium $x \in \mathcal{E}$ counted with algebraic multiplicity. Show, that the even/odd parity of n_x is the same, for all equilibria $x \in \mathcal{E}$.
Hint: Show that the tangent space $T_x M$ is invariant under the linearization.

Problem 34: Traveling wave solutions $u = u(x - ct)$ of

$$u_t = u_{xx} + f(u), \qquad u \in \mathbb{R}^N,\ f \in C^2,\ f \text{ dissipative},$$

with large wave speed $c \gg 1$ are solutions of the reduced flow

$$u' = -c^{-1}\tilde{f}(u),$$

where $\tilde{f}$ is a small perturbation of f. Prove a similar result for $c \ll -1$. How do solutions for positive and negative c relate to each other, in the reduced equation? What does this relation imply for the traveling wave solutions?

Problem 35: Find traveling wave solutions for the KPP equation

$$u_t = u_{xx} + u(1 - u).$$

Problem 36: How does the dimension of the inertial manifold of the equations

(i) $\varepsilon^2 u_t = u_{xx} + f(u)$;

(ii) $u_t = \varepsilon^2 u_{xx} + f(u)$;

(iii) $u_t = u_{xx} + \varepsilon^2 f(u)$;

where $0 < x < 1$, $f(u) \in BC^2$ with $uf(u) < 0$ for large $|u|$ and Dirichlet boundary conditions change with $\epsilon \searrow 0$.