Homework assignment

Infinite-Dimensional Dynamical Systems

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Due Tuesday, July 12, 2016, 10:15

Problem 41: Let $v \in \mathbb{R}^N$ be a hyperbolic equilibrium of some ODE $\dot{u} = f(u)$ with stable and unstable manifolds W^s, W^u.

(i) Are W^s, W^u transverse at v ?

(ii) Assume $W^s \cap W^u \neq \{v\}$. Show that v possesses a homoclinic orbit $u(t)$, i.e. $u(t) \to v$ for $t \to \pm\infty$. Can W^s and W^u be transverse along $u(t)$?

Problem 42: Show $z(\Phi_n) = n$ for the n–th eigenfunction $\Phi_n(x)$ of Sturm-Liouville eigenvalue problems

$$\Phi''(x) + b(x)\Phi'(x) + c(x)\Phi(x) = \mu\Phi(x),$$

on the unit interval with Neumann boundary conditions and for real eigenvalues μ. In other words, show that $\Phi(x)$ possesses exactly n simple zeros.

Hint: Consider polar coordinates $\Phi + i\Phi' = r\exp(i\varphi)$ and study how the angle φ depends on x and μ.

Problem 43: Determine the Sturm permutations, according to Fusco and Rocha, which arise from the Chafee-Infante problem $u_t = u_{xx} + \lambda u(1 - u^2)$ with Neumann boundary conditions.
How would you proceed, "analogously", for Dirichlet boundary conditions? How would your result change?

Problem 44: Consider the problem

$$u_t = u_{xx} + f(u)$$

on the unit interval with Neumann boundary conditions. Show that the Sturm permutation σ is an involution, i.e. $\sigma^2 = \mathrm{id}$. Can you discover more general classes of nonlinearities $f = f(x, u, u_x)$ such that σ is necessarily an involution?