

1. What is a semigroup and when do we say it's cpt?
2. Recall the def. of a prehistory of $x \in X$ and define the invariance of $I \setminus x$.
3. Define $w(x)$ for $x \in X$ and $w(H)$, $H \subset X$.
When do we say that $A \subset X$ attracts $B \subset X$ under semigroup $T(t)$?
4. Assume $T(t)$ be cpt semigroup, $\gamma^+(H)$ bounded.
What can you conclude about $w(H)$?
Prove or disprove: H connected $\Rightarrow w(H)$ connected
5. When do we say that the semigroup T is (point, cpt, bounded) dissipative?
6. Let $T(t)$ be cpt, dissipative semigroup.
Prove that the maximal cpt invariant set A Fullfills $A = \{ \text{bounded, external sol} \}$?
7. Define what a Lyapunov set is and give one example.
8. Give the def of (Topological / Hausdorff / Box-counting) dimension.
9. Prove $\dim_H(k) \leq C(k)$
10. Under which assumption we can state
 $C(k) \leq N \frac{\log(1 + 2^{1/\delta})}{\log(1/\delta)} < \infty$
11. State Mañé's thm. In the setting of Mañé's thm
What assumption do we need to have the bound.
12. Prove: f cpt $\Rightarrow f'$ cpt
13. Let k be cpt metric, $n(\epsilon) :=$ minimal # of ϵ -balls required to cover k .
Assume $\exists 0 < \eta < \alpha < 1$ s.t. $n(\eta\epsilon) \leq \alpha^n n(\epsilon)$ ($\forall 0 < \epsilon \leq \epsilon_0$)
Prove $C(k) \leq N \frac{\log \alpha}{\log(\eta/\alpha)}$
14. Consider the setting: $\dot{x}(t) = f(x_t)$, $t \geq 0$
 $x_0 = \varphi \in X = C^0([-\tau, 0], \mathbb{R}^n)$
 $x_t(w) = x(t+w)$
Suppose linear growth and prove $T(\tau)$ is cpt.
15. Consider again the setting of 14.
Prove the following statements and conclude
(i) $\|x_t\| \|x(t)\| \leq C_\tau(\varphi)$
(ii) $\exists \tilde{C}$, independent of $\|\varphi\|$ s.t. $\|w(\varphi)\| \leq \tilde{C}$

16. Consider the Parabolic equation

$$u_t = u_{xx} + f(x, u), \quad \{0 < x < 1\} =: \Omega, f \in C^2(L^\infty, \text{Lip})$$

$$u = 0 \quad \text{at } x = 0, 1$$

$$u = u_0(x) \quad \text{at } t = 0$$

Consider the Parabolic equation prove $T(t)$ is dissipative

17. Define the Lyapunov dimension and explain all components

18. Give the def. of determining modes/nodes and feedback vertex set

19. Give the def of an Inertial Manifold

20. Formulate the Inertial Manifold thm.

21. Provide one example of a setting where no C^1 Inertial Manifold exists

22. What is a zero number in the setting of Sturm and what is the main property.

23. How can Anagnin's lemma be extended to the general nonlinear case. Prove it!

24. Consider a hyperbolic equilibrium v in the setting of Sturm.

How are the zero number of solutions in $W^s(v)/W^u(v)$ and the unstable $\dim W^u(v)$ related? Sketch why this holds

25. State the cascading lemma and give the definition of the connection graph

26. In Sturm's setting. Explain what a Sturm permutation is.

27. When do we say an equilibrium blocks heteroclinics in between two other equilibria? State the "Liberation" thm.

28. State the thm on structural stability of Sturm attractor.

29. Under which cond is a Sturm permutation realizable as an attractor.

