1. Exercise for Differential Equations I (SS 2016 Väth)

Please finish until: Friday, April 29, 2016

Exercise 1. Which of the following equations is an ODE or can be transformed into one? (And if this is the case: how?) Which of those are explicit, possibly after some transformation?

a) $\frac{t}{x(t)} + x'(t) = 0$ (1 point)
b) $\sin(x'(t)) = tx(t)$ (1 point)
c) $\sin(x(t)) = tx'(t)$ (1 point)
d) $x'(t) = x(t-1)$ (1 point)
e) $x'(x(t)+t) = t$ (1 point)
f) $\frac{dx(t,s)}{dt} = 2s^2t^2x(t,s)^2$ (2 points)
g) $x(t) = \int_0^t sx(s)^2\,ds$ (3 points)
h) $x(t) = \int_0^t tx(s)^2\,ds$ (4 points)

Exercise 2. Write explicitly the system associated to the 2nd order equation $\ddot{x} = \alpha x$ and sketch the phase portrait in the cases $\alpha = -2, 0, 1$. In particular, in which cases do you obtain closed trajectories? $(1+3+1+3$ points)

Exercise 3. Draw some isoclines of $\dot{x} = t + \sin x$ and sketch some solution curves. (3 points)

Exercise 4. Let $T \in \mathbb{R}$ be fixed. A theorem of the lecture implies: A function x solves an autonomous (explicit or implicit) system on some interval I if (and only if) $y(t) = x(t+T)$ solves this system on the interval $I - T$.

a) Formulate and prove a variant of this assertion for nonautonomous systems. (2 points)
b) Formulate an analogous assertion for the time inversion $y(t) = x(-t)$ (for implicit systems of order n). (2 points)
c) Answer the preceding question (time inversion) in particular for explicit autonomous systems of first order. (2 points).

Exercise 5. Prove Theorem 1.2. (2 points + 3 extra points + 3 points)