

1. Exercise for Differential Equations II (WS 2016/17, V ath)

Time limit: Wednesday, October 26, 2016

Exercise 1. Let $M \subseteq \mathbb{R}^n$ be open, $f \in C^1(M, \mathbb{R}^n)$, and Φ be the flow associated to $x' = f(x)$. Show that $\Phi(k, \cdot)$ ($k \in \mathbb{Z}$) is the discrete flow associated to the diffeomorphism $F = \Phi(1, \cdot)$. Does every diffeomorphism F arise in such a way? (4 + 6 Points)

Exercise 2. Let $M, N \subseteq \mathbb{R}^n$ be open, and $f \in C^1(M, \mathbb{R}^n)$, $g \in C^1(N, \mathbb{R}^n)$: Let Φ and Ψ denote the flows associated to the dynamical systems $x' = f(x)$ and $y' = g(y)$, respectively. Show that for a diffeomorphism $\varphi: M \rightarrow N$ the following assertions are equivalent. (8 Points)

1. Φ and Ψ are conjugate via the corresponding diffeomorphism φ .
2. For each solution x of $x' = f(x)$ the function $y(t) = \varphi(x(t))$ is a solution of $y' = g(y)$.
3. For every $u \in M$

$$\varphi'(u)f(u) = g(\varphi(u)).$$

4. For every $v \in N$

$$g(v) = \varphi'(\varphi^{-1}(v))f(\varphi^{-1}(v)).$$

Exercise 3. Let $M \subseteq \mathbb{R}^n$ be open, $f \in C^1(M, \mathbb{R}^n)$, and Φ be the flow associated to $x' = f(x)$. Is it possible to obtain f from Φ ? How? Use this again to show that the formula of the previous exercise is *necessary* for the conjugacy of Φ and Ψ . (4 Points)