Homework Assignments

Dynamical Systems I

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http://dynamics.mi.fu-berlin.de/lectures/

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Problem 13: A (point-sized) person Pu —trying to escape from Spain — starts from his office in Barcelona, which happens to be located at the origin $(x, y) = (0, 0)$ of the plane $\mathbb{R}^2$. He runs along the positive x-axis with speed 1. At the same moment a (point-sized) police officer Po starts at Madrid $(x, y) = (0, 3)$ and chases Pu. He has the same speed 1 as Pu, and always runs directly towards Pu.

Will Po ever catch Pu? Or else, how close does Po get?

Remarks: Any similiarities with living or dead persons are more or less coincidental. For mathematical purposes, the earth is unbounded and flat. Neither Po nor Pu ever rest. Use appropriate coordinates (e.g. r = distance between Pu and Po, φ = angle of the x-axis with the connecting line of both persons) and solve the resulting system by separation of variables.

Extra credit: Is always running towards Pu the best strategy for Po? If not, which one is optimal?

Problem 14: Consider the two scalar vector fields

(i) $\dot{x} = f(x) := x \in \mathbb{R}$,

(ii) $\dot{\xi} = g(\xi) := (1 + \xi^4)\xi \in \mathbb{R}$.

Prove or disprove that f-orbits and g-orbits coincide.

Remark: The theorem stated in the lecture is not applicable because the flow of (ii) is not global.

Problem 15: Solve the following initial-value problems by separation of variables and determine the maximal forward and backward time intervals of existence of the solutions:

(i) $\dot{x} = x^2 e^{-t}, \qquad x(0) = 1,$

(ii) $\dot{x} = 1 + x^2, \qquad x(0) = 1/2,$

(iii) $\dot{x} = x(2 - x), \qquad x(0) = 1,$

(iv) $\dot{x} = -x^2 e^{1/x}, \qquad x(0) = 1.$

Problem 16: Consider a radially symmetric C^1 vector field in the (x, y)-plane,

$$\begin{aligned} \dot{x} &= f(x^2+y^2)x - g(x^2+y^2)y, \\ \dot{y} &= g(x^2+y^2)x + f(x^2+y^2)y. \end{aligned}$$

(i) Find an Euler multiplier $\mu = \mu(x^2+y^2)$ to obtain a divergence-free vector field.

(ii) Find an example of the above form that does *not* possess any nontrivial first integral.

(iii) What is wrong?