Homework Assignments

Dynamical Systems I

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http://dynamics.mi.fu-berlin.de/lectures/

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Problem 25: Prove or disprove: The linearization of a flow $\Phi_t(x_0)$, at an equilibrium $x_0 = 0$, is the flow of the linearized vector field, at $x_0 = 0$.

Problem 26: Consider the evolution $x(t){=}\Phi_{t,0}(x_0)$ of the scalar differential equation

$$\dot{x} = x(x + \sin t), \qquad x(0) = x_0.$$

(i) Show that the linearization $v(t){:=}D_{x_0}\Phi_{t,0}(0)$ remains bounded, for $t \to \infty$.

(ii) For $x_0 > 0$, show that the sequence $x_n{:=}x(2\pi n)$ increases strictly with n.

(iii) Show that (ii) implies $x(t) > 1$, for all $t > t_0(x_0)$.

(iv) Conclude $x(t) \to \infty$, for $t \to \infty$ and arbitrarily small $x_0 > 0$.

How do the contradictory results (i) and (iv) relate to the differentiability theorem from the lecture?

Problem 27: Consider the Banach space BC^1 of continuously differentiable vector fields $f\colon X \to X = \mathbb{R}^N$ with

$$\|f\|_{BC^1} := \sup_{x\in X}(|f(x)| + |f'(x)|) < \infty.$$

Let f, g be vector fields in BC^1 and let $x(t){=}\Phi_t^f(x_0)$ denote the flow of

$$\dot{x}(t) = f(x(t)), \qquad x(0) = x_0\,.$$

Is the map

$$\Psi:\ BC^1 \to X, \qquad f \mapsto \Psi(f) := \Phi_t^f(x_0),$$

differentiable with respect to $f \in BC^1$, for fixed t? If so, then which differential equation is solved by the partial derivative $v(t) := \mathrm{D}_f\Phi_t^f(x_0)g$?

Problem 28: Let $x(t) = \Phi_{t,t_0}(x_0)$ denote an evolution on $\mathbb{R}^N$, and let $f = f(t, x)$ denote the associated nonautonomous vector field. Assume f is continuously differentiable. Which differential equation, and which initial condition at $t = t_0$, does $v(t) := \partial_{t_0} \Phi_{t,t_0}(x_0)$ satisfy?