Homework Assignments

Dynamical Systems I

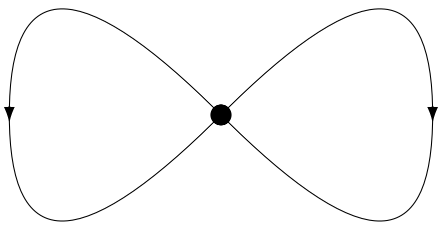
Bernold Fiedler, Hannes Stuke

http://dynamics.mi.fu-berlin.de/lectures/

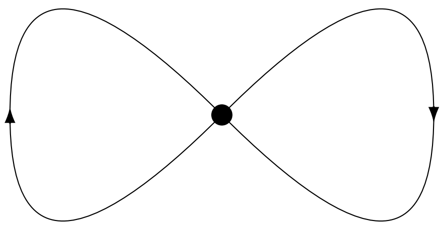
due date: Friday, July 13, 2018

Problem 41: Which of the following sets are possible ω-limits of a (single) trajectory of some planar flow? Which of the sets cannot occur as ω-limits (of a single trajectory)? Justify your claims, without providing explicit vector fields.

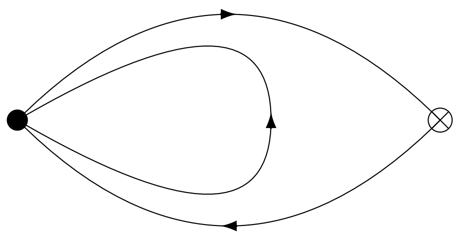
(i)



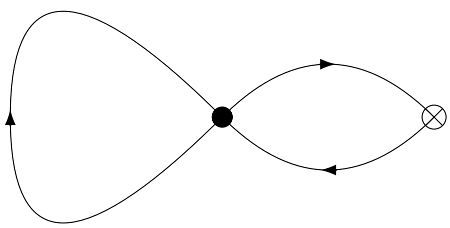
(ii)



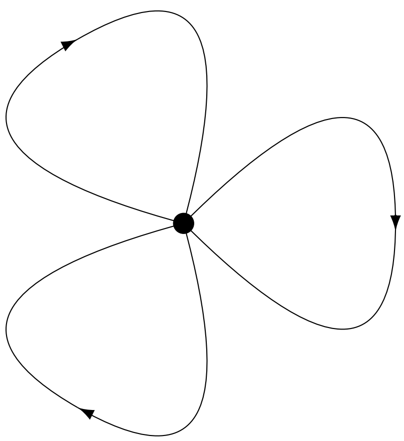
(iii)



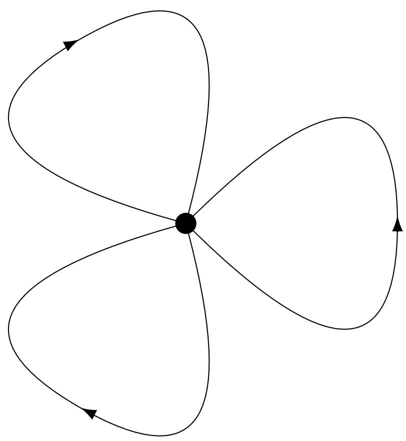
(iv)



(v)



(vi)



Discs ● denote equilibria (of any type) and crossed out circles ⊗ denote hyperbolic saddles.

Problem 42: Prove or disprove the theorem of Poincaré & Bendixson for flows on

(i) the sphere S^2,

(ii) the torus T^2.

Problem 43: Prove or disprove a discrete version of Poincaré & Bendixson in the plane. Consider

$$x_{n+1} = f(x_n), \qquad x_n \in \mathbb{R}^2,$$

where $f \in C^1$ is a diffeomorphism. Define the discrete flow $\phi_n(x_0)$ by $\phi_n(x_0) := f^n(x_0)$. Let the forward trajectory $\gamma_+(x_0) := \{\phi_n(x_0),\ n \in \mathbb{N}\}$ of x_0 be bounded. Then $\omega(x_0)$ contains a periodic orbit or a fixed point.

Problem 44: Show that the system

$$\begin{aligned}\dot{x} &= x + y - (2x^2 + y^2)x,\\ \dot{y} &= -x + y - (2x^2 + y^2)y,\end{aligned}$$

possesses a periodic orbit.

Problem 45: [10 Extra points]
Let f be a differentiable vector field on $\mathbb{R}^3$. Prove or disprove: A trajectory coincides with its ω-limit set if, and only if, the trajectory is an equilibrium or a periodic orbit.
For your convenience and warning - There is an incorrect proof related to the claim, see H. Amann - Ordinary Differential Equations.