

Dynamical Systems 1: basic questions

31. When do we call a set M positively invariant, negatively invariant, or invariant with respect to a given flow Φ_t ?
32. Let the forward orbit $\gamma^+(x_0)$ of x_0 under a flow in \mathbb{R}^N be bounded. Is the ω -limit set of x_0
- (a) open,
 - (b) closed,
 - (c) bounded,
 - (d) compact,
 - (e) stable,
 - (f) asymptotically stable,
 - (g) unstable?

Why?

33. Let the forward orbit $\gamma^+(x_0)$ of x_0 under a flow in \mathbb{R}^N be bounded. Is the ω -limit set of x_0
- (a) nonempty,
 - (b) finite,
 - (c) discrete,
 - (d) connected,
 - (e) positively invariant,
 - (f) negatively invariant,
 - (g) invariant?

Why?

34. How is a Lyapunov function of a vector field defined? When do we call a Lyapunov function strict?
35. How does a Lyapunov function restrict possible ω -limit sets of a flow?
36. Formulate the invariance principle of LaSalle.
37. How are stability and asymptotic stability of a nonempty, compact, invariant set defined?
38. Why can the Brusselator

$$\dot{x}_1 = a - x_1 - bx_1 + x_1^2x_2, \quad \dot{x}_2 = bx_1 - x_1^2x_2, \quad a, b > 0,$$

possess a nonstationary periodic orbit? Can it possess a *stable* nonstationary periodic orbit?

39. Formulate the theorem of Poincaré & Bendixson on ω -limit sets of bounded orbits of planar vector fields.