Homework Assignments

Dynamical Systems II

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http://dynamics.mi.fu-berlin.de/lectures/

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Problem 1: [Discrete Floquet] Prove or disprove: There exist matrices A_i, $i = 0, 1$, having all eigenvalues in the interior of the complex unit disk, such that the iteration

$$x_{n+1} = A_{n (\mathrm{mod}\, 2)} x_n,$$

is unstable.

Problem 2: (a) Consider the non-autonomous linear system $\dot{y}(t) = A(t)y(t)$, with periodic matrix A, $A(t+p) = A(t)$. For given initial time t_0, the Floquet theorem yields a solution $y(t) = W(t, t_0)y(t_0)$ of the form

$$W(t, t_0) \;=\; Q_{t_0}(t)\mathrm{e}^{B_{t_0} t},$$

with constant matrix B_{t_0} and periodic matrix Q_{t_0}, $Q_{t_0}(t+p) = Q_{t_0}(t)$. How do the matrices B_{t_0} and Q_{t_0} depend on t_0?

(b) Consider the autonomous vector field $\dot{x} = f(x)$. Let S be a Poincaré section to a given periodic orbit γ of this system. Prove that the Floquet multipliers of γ are independent of the choice of the Poincaré section S.

Problem 3: Let $I \subset \mathbb{R}$ be an interval and $A \in C^1(I, \mathbb{R}^{n\times n})$.

Prove: If A and $\dot{A}$ commute, i.e. if $[A(t), \dot{A}(t)] := A(t)\dot{A}(t) - \dot{A}(t)A(t) = 0$ for all $t \in I$, then

$$\frac{\mathrm{d}}{\mathrm{d}t} e^{A(t)} \;=\; \dot{A}(t)\, e^{A(t)} \;=\; e^{A(t)}\, \dot{A}(t).$$

Problem 4: Floquet theory tells us that we can decompose the Wronskian $W(t, s)$ of a T-periodic linear differential equation as

$$W(t, 0) = Q(t)\mathrm{e}^{Bt}, \qquad Q(t+T) = Q(t),\ Q(0) = \mathrm{Id}.$$

Is it always possible to find a decomposition of the form

$$W(t, 0) = \mathrm{e}^{Ct} R(t), \qquad R(t+T) = R(t),\ R(0) = \mathrm{Id}?$$