Homework Assignments

Dynamical Systems II

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http://dynamics.mi.fu-berlin.de/lectures/

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Problem 5: Consider the map $A : \mathbb{R} \to \mathbb{R}$,

$$A(y) \;=\; \begin{cases} 2y, & 0 \leq y < 1 \\ A(y-1)+1, & 1 \leq y \\ A(y+1)-1, & y < 0 \end{cases}$$

Thus $A(y+1) = A(y)+1$ for all y and A defines a map $\tilde{A} : S^1 \to S^1 = \mathbb{R}/\mathbb{Z}$. However A and $\tilde{A}$ are not homeomorphisms. Nonetheless, try to define the usual “rotation number” $\varrho(y_0)$ for initial conditions y_0. Does $\varrho(y_0)$ depend on y_0?

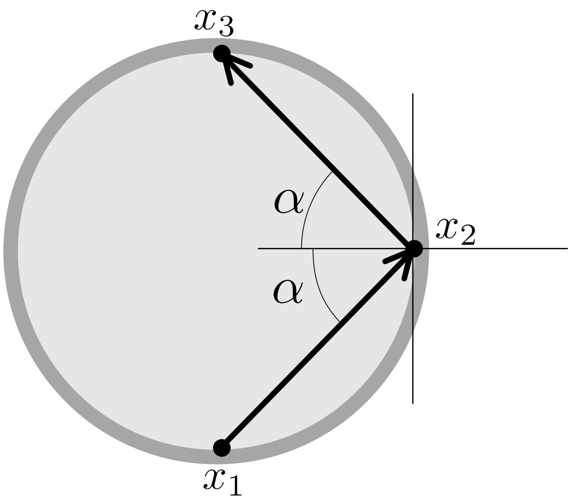
Problem 6: Consider the pendulum $\ddot{x} + f(x) = 0$ with odd nonlinearity $f \in C^1$ and $f' \geq 1$. Let $p(a) > 0$ denote the minimal period of the periodic orbit with amplitude $a > 0$.
Prove or disprove: The trivial Floquet multiplier 1 is:

(i) always algebraically double,

(ii) geometrically simple for $p'(a) \neq 0$.

Hint: Consider Liouville's theorem for the two dimensional system.

Problem 7: [Billiard] An ideal point-sized billiard ball moves with constant speed on the unit disc $B^2 \subset \mathbb{R}^2$. Fix an initial position x_1 on the boundary and a shooting angle α. What is the ω-limit set of the sequence of collision points $\{x_n\}_{n\in\mathbb{N}}$ in S^1 depending on α?



The ω-limit set of $\{x_n\}_{n\in\mathbb{N}}$ is defined as

$$\omega(\{x_n\}_{n\in\mathbb{N}}) := \{y \in S^1 | \text{ there exists a subsequence } n_k \xrightarrow{k\to\infty} \infty \text{ for which } x_{n_k} \to y\}$$

Problem 8: Let $\tilde{A} : S^1 \to S^1$ be a homeomorphism of the circle that reverses orientation, i.e. the induced map $A : \mathbb{R} \to \mathbb{R}$ on the covering space $\mathbb{R}$ satisfies $A(x+2\pi) = A(x) - 2\pi$ for all $x \in \mathbb{R}$.

Prove or disprove: $\tilde{A}$ has a fixed point.