Homework Assignments

Dynamical Systems II

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http://dynamics.mi.fu-berlin.de/lectures/

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Problem 9: [counts as 2 problems] Let a, b denote orientation-preserving $\mathcal{C}^2$-diffeomorphisms of S^1 and ϱ the rotation number. Prove or disprove:

(i) $\varrho(a) + \varrho(a^{-1}) = 0$;

(ii) $\varrho(aba^{-1}) + \varrho(b^{-1}) = 0$;

(iii) if $ab = ba$ and $\varrho(a) \notin \mathbb{Q}$ then $\varrho(ab) = \varrho(a) + \varrho(b)$;

(iv) if $\varrho(a) = \varrho(b) \notin \mathbb{Q}$ then $ab = ba$;

(v) if $\varrho(a) = \varrho(b) \notin \mathbb{Q}$ then $\varrho(ab) = 2\varrho(a)$;

(vi) $\varrho(ab) = \varrho(ba)$;

(vii) if $n \in \mathbb{Z}$ then $\varrho(a^n) = n\varrho(a)$;

(viii) $\varrho(ab) = \varrho(a) + \varrho(b)$.

Problem 10: Let $0 < \beta < 1$ be irrational and

$$s_n := \operatorname{sign}(\sin(n\pi\beta)), \qquad n = 1, 2, 3, ...$$

The sequence

$$w_n := |s_n - s_{n+1}|/2$$

detects the sign changes of the sequence s_n. Prove:

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} w_n = \beta.$$

Free extra: Is it possible to recover a rational number β from the sequence s_n?

Problem 11: Calculate the rotation number $\varrho(\alpha)$ of the time-2π-map of the differential equation

$$\dot{x} = \alpha + \sin(x - t), \qquad x \in S^1.$$

Problem 12: Find an example to show that a homeomorphism f of the circle with rational rotation number $\varrho(f)$ is not necessarily conjugated to a rigid rotation by the angle $\varrho(f)$.