

Homework Assignments

Dynamical Systems II

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<http://dynamics.mi.fu-berlin.de/lectures/>

due date: **Friday, November 16, 2018, 12:00**

Problem 13: Let A be a C^2 circle diffeomorphism with irrational rotation number. Consider the set \mathcal{H} of circle homeomorphisms which conjugate A to a rigid rotation. Endow \mathcal{H} with the sup-norm distance. Show that \mathcal{H} is (homeomorphic to) a circle.

[Extra credit: The rational case]

Let A be a C^2 circle diffeomorphism with rational rotation number and assume that all points are periodic. Describe the set \mathcal{H} of circle homeomorphisms which conjugate A to a rigid rotation.

Problem 14: Consider a nonlinear flow on \mathbb{T}^2 with the y -axis S^1 as global Poincaré section. Let $\tilde{\Phi}_t$ denote the flow after multiplication by an Euler multiplier which fixes the Poincaré time to be identically 2π . Assume that the rotation number ρ of $\tilde{\Phi}_{2\pi}$ is irrational. Show that there exists a homeomorphism H of \mathbb{T}^2 which conjugates $\tilde{\Phi}_t$ to the parallel flow

$$\begin{aligned} \dot{x} &= 1, \\ \dot{y} &= \rho. \end{aligned}$$

Problem 15: Consider the Fibonacci iteration on the torus

$$\begin{aligned} A : \mathbb{T}^2 &\rightarrow \mathbb{T}^2, \\ (x, y) &\mapsto (y, x + y) \pmod{1}. \end{aligned}$$

Is it well-defined? Is it invertible? Calculate stable and unstable manifolds of the fixed point $(0, 0)$ under the iteration A^n . Are they dense on the torus?

Problem 16:

Consider a matrix $B \in GL(\mathbb{R}^N)$ with eigenvalues $\lambda_1 > \dots > \lambda_N > 0$, $N = 3$. B induces a map on the sphere S^{N-1} given by

$$\begin{aligned}\tilde{B} : S^{N-1} &\rightarrow S^{N-1} \\ \varphi &\mapsto \frac{B\varphi}{\|B\varphi\|}.\end{aligned}$$

Given the iteration $\varphi_{n+1} = \tilde{B}\varphi_n$:

- (i) Determine all fixed points.
- (ii) Show that they are hyperbolic.
- (iii) Determine the stable and unstable manifolds W^s and W^u and the heteroclinic orbits between the fixed points.

[Extra credit] What about the general case $N > 3$.