

Homework Assignments

Dynamical Systems II

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Problem 21: Given $2 \leq N \in \mathbb{N}$, consider the shift space on N symbols $S := \{s = (s_k)_{k \in \mathbb{Z}} \mid s_k \in \{0, \dots, N-1\}\}$ together with the cylinder metric topology induced by

$$d(x', x) = \sum_{k \in \mathbb{Z}} \frac{1}{2^{|k|}} |x'_k - x_k|.$$

Prove that S is a Cantor set, i.e:

- (i) S is totally disconnected.
- (ii) S is perfect. In other words, $s \in \text{clos}(S \setminus \{s\})$ for any $s \in S$.
- (iii) S is compact.

Problem 22: Calculate all fixed points of the bouncing-ball map f :

$$\begin{aligned}\Phi_{j+1} &= \Phi_j + v_j, \\ v_{j+1} &= \alpha v_j - \gamma \cos(\Phi_j + v_j),\end{aligned}$$

with $\Phi_j \in S^1 = \mathbb{R}/(2\pi\mathbb{Z})$ and $v_j \in \mathbb{R}$, for $0 < \alpha < 1$ and $0 < \gamma$. How many fixed points does f have, for given α, γ ? Determine the type (hyperbolically stable, hyperbolically unstable, non-hyperbolic) of the fixed points. Sketch the dependence of the fixed points on γ , for $\alpha = \frac{1}{2}$. What happens for $\alpha \rightarrow 1$?

Problem 23: Let Φ be a Smale horseshoe homeomorphism with alphabet A , invariant set I , and shift space S on A . Let τ denote a homeomorphism which conjugates Φ on I to the shift σ on S .

Let τ' also be a homeomorphism which conjugates Φ on I to the shift σ on S . Are τ and τ' identical? If not, how would they differ?

Problem 24: In the setting of Problem 23, for $s \in S$, let $V(s)$ denote the local stable leaf of $p := \tau(s)$.

a) Describe the global stable set $W^s(p)$ of p , i.e. the set of q for which

$$\Phi^k(q) \in \bigcup_a V_a \text{ and } \lim_{k \rightarrow +\infty} \text{dist}(\Phi^k(q), \Phi^k(p)) = 0.$$

b) Let V denote the union of local stable leaves $V(s')$, with $s' \in S$. Show that V is compact.

c) Show that V is the stable set of I , i.e. V consists of all q with $\Phi^k(q) \in \bigcup_a V_a$ and ω -limit set in I , under iteration of Φ .

d) Is $W^s(p)$ dense in the stable set V of I ?