Homework Assignments

Dynamical Systems II

Bernold Fiedler, Hannes Stuke

http://dynamics.mi.fu-berlin.de/lectures/

due date: Friday, December 21, 2018, 12:00

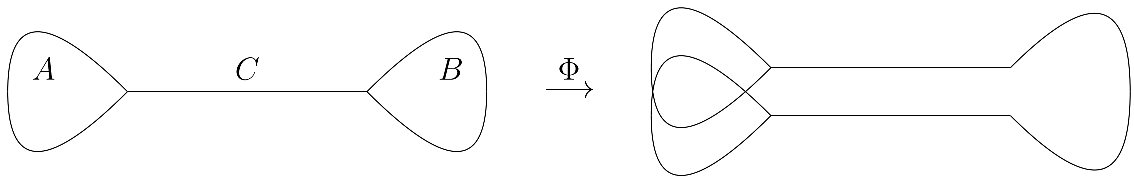
Problem 33: [Grobman & Hartman]
Consider the map $\Phi(x) = Ax + f(x) : \mathbb{R}^2 \to \mathbb{R}^2$ with A a hyperbolic matrix. Show that there exists $\varepsilon > 0$ such that for every function $f \in B_\varepsilon(0)$

$$B_\varepsilon(0) := \left\{ g \in C^0(\mathbb{R}^2, \mathbb{R}^2) \mid g \text{ bounded and } \sup_{(x,y)\in\mathbb{R}^2} \frac{|g(x)-g(y)|}{|x-y|} < \varepsilon \right\}$$

there is a homeomorphism H of $\mathbb{R}^2$ conjugating A to Φ.

Problem 34: Try to realize the following graph by a continuous map $\Phi : \mathbb{R}^2 \to \mathbb{R}^2$ (similar to the case of the Plykin attractor)

$$\begin{aligned} A &\to A \\ B &\to A \\ C &\to C + B - C \end{aligned}$$



(i) Why does this construction not yield a hyperbolic attractor?

(ii) Can Φ contain a horseshoe?

Problem 35: Consider 2×2 matrices $A \in \mathrm{SL}(2, \mathbb{Z})$ with integer coefficients and determinant 1. The linear map A on $\mathbb{R}^2$ preserves orientation and area.

(i) Prove that every $A \in \mathrm{SL}(2, \mathbb{Z})$ defines a diffeomorphism of the torus $T^2 = \mathbb{R}^2/\mathbb{Z}^2$.

(ii) Is $A = \mathrm{id} \in \mathrm{SL}(2, \mathbb{Z})$ structurally stable?

Extra credit: Which $A \in \mathrm{SL}(2, \mathbb{Z})$ are structurally stable?

Problem 36: Consider the iteration on the 2-torus $T = (\mathbb{R}/\mathbb{Z})^2$ defined by the matrix

$$B = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}.$$

Find a horseshoe for a suitable iterate B^k, $k > 0$.

Hint: Identify the torus with the unit square $[-1/2, 1/2)^2$ and investigate the images of a parallelogram parallel to the eigenvectors of B.