Homework Assignments

# Dynamical Systems II

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http://dynamics.mi.fu-berlin.de/lectures/

**due date: Friday, January 18, 2019, 12:00**

**Problem 37:** Discuss the existence of global center manifolds $M^c(0)$ tangent to the center eigenspace at 0,

$$M^c(0) := \left\{ x \in \mathbb{R}^N \mid \sup_{t\in\mathbb{R}} |\pi^h \Phi_t x| < \infty \right\},$$

for the following systems of ODEs:

$$\text{(a)} \left\{ \begin{array}{rcl} \dot{x} &=& x^2 + y, \\ \dot{y} &=& -x^2 - y, \end{array} \right. \qquad \text{(b)} \left\{ \begin{array}{rcl} \dot{x} &=& 2\exp\left(\frac{-1}{x^2+y^2}\right), \\ \dot{y} &=& 2\exp\left(\frac{-1}{x^2+y^2}\right) + \arctan y, \end{array} \right.$$

$$\text{(c)} \left\{ \begin{array}{rcl} \dot{x} &=& y, \\ \dot{y} &=& -x, \\ \dot{z} &=& x^2 + y^2 + z. \end{array} \right.$$

Justify your answer and if it is positive give an explicit formula for $M^c(0)$.

**Problem 38:** Consider the system of differential equations

$$\begin{array}{rcl} \dot{x} &=& xy, \\ \dot{y} &=& -y + x^2. \end{array}$$

Use a (local) center manifold to decide whether the equilibrium $x = y = 0$ is asymptotically stable.

*Hint:* Use the invariance of the center manifold to calculate the necessary terms of its Taylor expansion.

**Problem 39:** [Vanderbauwhede] Show that the analytic ODE

$$\begin{array}{rcl} \dot{x} &=& \mu x - x^2, \\ \dot{y} &=& y - x^2, \\ \dot{\mu} &=& 0 \end{array}$$

does not admit any analytic center manifold $M^c_{loc}(0)$ at the non-hyperbolic equilibrium $(0, 0, 0)$.

*Extra credit:* Can $M^c_{loc}(0)$ be $C^\infty$?

**Problem 40:** Consider the ODE $\dot{x} = f(x)$ with $f(0) = 0$. Assume that zero is an algebraically simple eigenvalue of the linearization $Df(0)$ and all other eigenvalues have nonzero real part.
Can there exist nontrivial periodic orbits in an arbitrarily small neighborhood of $x = 0$? What if $Df(0)$ has a pair of algebraically simple purely imaginary eigenvalues, instead, and all other eigenvalues have nonzero real part?