Homework Assignments

Dynamical Systems II

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http://dynamics.mi.fu-berlin.de/lectures/

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Problem 41: Let $M^c = \text{graph}\Psi$ be a C^1 center manifold for

$$\dot{x} = Ax + g(x), \text{ where } g \in C^1,\ g(0) = 0,\ Dg(0) = 0.$$

Here $\Psi : E^c \to E^h$ where $\mathbb{R}^N = E^c \oplus E^h$ is the eigenspace decomposition of A. Prove that M^c is tangential to E^c, i.e. $\Psi'(0) = 0$.

Problem 42: Let $R : \mathbb{R}^N \to \mathbb{R}^N$ be a linear involution ($R^2 = \text{id}$). Assume that the C^1 vector field f is equivariant under the symmetry R, that is

$$f \circ R \; = \; R \circ f.$$

(i) Let the assumptions of the theorem on the existence of a *global* center manifold be satisfied. Prove that the center manifold M^c inherits the symmetry of the vector field, that is

$$R(M^c) \; = \; M^c.$$

(ii) Assume $f(0) = 0$; let the assumptions of the theorem on the existence of a *local* center manifold at 0 be satisfied. Prove that there exists a symmetric local center manifold $M^c_{\text{loc}}(0)$, i.e.

$$R(M^c_{\text{loc}}(0)) \; = \; M^c_{\text{loc}}(0).$$

Problem 43: For linear maps A, B the usual commutator is defined as $[A, B] := AB - BA$. Show that

$$[\text{ad}\ A, \text{ad}\ B] = \text{ad}\ [A, B],$$

where $((\text{ad}\ A)f)(x) := Af(x) - f'(x)Ax$ for $f \in C^\infty(\mathbb{R}^N, \mathbb{R}^N)$.

Problem 44: Consider the space of homogeneous quadratic vector polynomials on $\mathbb{R}^2$

$$H_2(\mathbb{R}^2) := \operatorname{span}\left\{\begin{pmatrix} x^2 \\ 0 \end{pmatrix}, \begin{pmatrix} xy \\ 0 \end{pmatrix}, \begin{pmatrix} y^2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ x^2 \end{pmatrix}, \begin{pmatrix} 0 \\ xy \end{pmatrix}, \begin{pmatrix} 0 \\ y^2 \end{pmatrix}\right\}.$$

Determine the kernel of the map $\operatorname{ad}_2 A := \operatorname{ad} A|_{H_2(\mathbb{R}^2)}$ for the matrix

$$A := \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}.$$