Basic Questions of Dynamical Systems II

1. What is a Poincaré section to a periodic orbit of a flow?

2. What is a Poincaré map to a periodic orbit of a flow?

3. Formulate the Floquet theorem for non-autonomous, time periodic, linear differential equations.

4. Formulate the Floquet theorem for autonomous vector fields, linearized at a periodic orbit.

5. What are Floquet multipliers and Floquet exponents of periodic orbits of autonomous vector fields?

6. Why do periodic orbits of autonomous vector fields possess a trivial Floquet multiplier 1?

7. What is the lift $F : \mathbb{R} \to \mathbb{R}$ of a homeomorphism $f : S^1 \to S^1$. Is the lift unique?

8. How is the rotation number of an orientation preserving homeomorphism $f : S^1 \to S^1$ defined? Does the definition make sense for orientation reversing homeomorphisms?

9. How are existence and minimal periods of periodic points related to the rotation number of a homeomorphism $f : S^1 \to S^1$?

10. State the ergodic theorem for parallel flows on the 2-torus.

11. Formulate the theorem of Denjoy for C^2-diffeomorphisms $f : S^1 \to S^1$.

12. How are local/global stable and unstable manifolds of hyperbolic equilibria of autonomous vector fields defined? How do they relate to the local/global stable and unstable manifolds of the time-1 map of the flow?

13. Formulate the theorem on the existence of local stable and unstable manifolds for hyperbolic equilibria of vector fields.

14. Formulate the theorem on the existence of local stable and unstable manifolds for hyperbolic fixed points of diffeomorphisms.

15. Are local/global stable and unstable manifolds of hyperbolic equilibria unique? What are the tangent spaces to stable and unstable manifolds at the equilibrium?

16. What is the Bernoulli shift on N symbols? Define the shift space, its metric, and the shift map.

17. How can we construct
    (a) periodic orbits of every period
    (b) a dense set of periodic orbits
    (c) a dense orbit
    for the shift on 2 symbols?

18. How does the shift on 2 symbols illustrate recurrence, as well as sensitive dependence on initial conditions?

19. Formulate the C^0-theorem on Smale's horseshoe.

20. Formulate the C^1-theorem on Smale's horseshoe.

21. Sketch a horseshoe construction for the bouncing-ball map

$$\begin{aligned} \varphi_{k+1} &= \varphi_k + v_k, \\ v_{k+1} &= v_k - \gamma \cos(\varphi_k + v_k), \end{aligned}$$

    under a suitable assumption on γ.

22. Define hyperbolic structure for diffeomorphisms.

23. What are transverse homoclinic points of diffeomorphisms?

24. Formulate the λ-lemma.

25. How does a transverse homoclinic point give rise to shift dynamics? Sketch the relevant picture.

26. What is the Plykin attractor?

27. How is structural stability of a diffeomorphism defined?

28. Is the set of structurally stable diffeomorphisms of a compact manifold open in the C^1-topology? Is it dense?

29. State Anosov's theorem on structural stability of diffeomorphisms of the 2-torus.

30. Give at least two examples of structurally stable diffeomorphisms of the 2-torus.

31. What is a strange attractor? Sketch an example and list relevant properties.

32. How are local center manifolds for non-hyperbolic equilibria of autonomous vector fields defined?

33. Formulate the theorem on the existence of local center manifolds for non-hyperbolic equilibria of autonomous vector fields.

34. Formulate the theorem on existence of local center manifolds for non-hyperbolic fixed points of diffeomorphisms.

35. Under which assumptions on the autonomous vector field does a global center manifold to a non-hyperbolic equilibrium exist? Is the global center manifold unique?

36. Is the local center manifold to a non-hyperbolic equilibrium unique? What is the tangent space to a C^1 center manifold at the equilibrium?

37. How are the local center-stable and center-unstable manifolds for non-hyperbolic equilibria of nonautonomous vector fields defined? When do they exist?