

## Basic Questions of Dynamical Systems II

38. Consider two smooth vector fields  $f, g \in C^\infty(\mathbb{R}^N, \mathbb{R}^N)$  and a matrix  $A \in \mathbb{R}^{N \times N}$ . How is the Lie derivative of  $f$  in direction  $g$  defined? How is the Lie bracket of  $f$  and  $g$  defined? What is the adjoint operator  $\text{ad}A$ ?
39. Formulate the theorem on the normal form of a smooth vector field near an equilibrium via transposed of the linearization.
40. Given a smooth vector field  $f$  with linearization  $A$  at an equilibrium, assume that  $\text{ad}(A^T)$  has trivial kernel when restricted to the space of homogeneous polynomials of any degree  $m \geq 2$ . What is the normal form of  $f$  at the equilibrium?
41. Consider a smooth vector field  $f$  with linearization  $A$  at an equilibrium. Assume  $AA^T = A^T A$ . Which additional symmetry does the  $\text{ad}(A^T)$  normal form of  $f$  possess?
42. Consider a smooth vector field  $f$  with real diagonal linearization  $A$  at an equilibrium. When do resonances occur? How are they related to the normal form of  $f$ ?
43. What is the normal form for Hopf bifurcation in the plane?