Basic Questions of Dynamical Systems II

44. What is a generic subset of a metric space $\mathcal{F}$? Give a sufficient condition on $\mathcal{F}$, such that the intersection $\bigcap_{i\in\mathbb{N}} \mathcal{G}_i$ of countably many generic subsets $\mathcal{G}_i$ of $\mathcal{F}$ is again a generic subset of $\mathcal{F}$. Give three examples of generic subsets of $\mathbb{R}^2$.

45. Let $\mathcal{C} \subset \mathbb{R}$ be a *Cantor set*, i.e. $\mathcal{C}$ is closed, totally disconnected, and any element of $\mathcal{C}$ is an accumulation point of other elements. Why is $\mathcal{C}$ uncountable?

46. When do we say that a bounded linear operator A between two Banach spaces X and Y is Fredholm? How is the Fredholm index $i(A)$ defined? When do we say that a nonlinear map $\varphi \in C^1(X, Y)$ is Fredholm?

47. Let X and Y be two Banach spaces and consider a map $\varphi \in C^r(X, Y)$, $r \geq 1$. What is a regular value of φ? Formulate the Smale-Sard Theorem on regular values.

48. Let X and Y be two Banach spaces and consider a map $\rho \in C^1(X, Y)$. Given a subset $S \subset Y$, when do we say that ρ is transverse to S:

    (a) at $y \in S$?

    (b) on $K \subset S$?

    (c) everywhere on S?

49. Formulate the Thom Transversality Theorem.

50. Formulate the Takens Embedding Theorem. How does it relate to the Whitney embedding theorem?