Homework Assignments

# Dynamical Systems III-Delay Differential Equations

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http://dynamics.mi.fu-berlin.de/lectures/

**due date: Wednesday, April 17, 2019, 12:00**

**Problem 1:** Consider the scalar delay differential equation

$$\dot{x}(t) = -x(t - \frac{\pi}{2}).$$

(i) Use the method of steps seen in the lecture to compute the solution $x(t)$ with initial condition $x_0(\theta) = \frac{\pi}{2} + \theta$, $\theta \in [-\frac{\pi}{2}, 0]$, for $t \in [0, \pi]$.

(ii) Look for solutions of the form $\exp(\lambda t)$ with $\lambda \in \mathbb{C}$ and show that the equation has periodic solutions. How does this differ from the behavior of the analogue equation without delay $\dot{x}(t) = -x(t)$?

**Problem 2:** Consider the differential delay equation

$$\dot{x}(t) = f(x(t-1)), \tag{1}$$

where $f \in C^\infty(\mathbb{R}, \mathbb{R})$. Let $\phi \in C \coloneqq C^0([-1, 0], \mathbb{R})$ be an initial condition yielding a solution $x^*(t)$ to the equation (1) that exists for every $t \geq 0$, i.e.

$$x^*(t) = \phi(t),\ -1 \leq t \leq 0 \text{ and } \dot{x}^*(t) = f(x^*(t-1)),\ t > 0.$$

Define $x_t^* \in C$ as $x_t^*(\theta) \coloneqq x^*(t + \theta)$, $\theta \in [-1, 0]$, $t \geq 0$. Show that

(i) The regularity of the solutions improves with time, i.e. $k \leq t \leq k + 1,\ k \in \mathbb{N} \Rightarrow x_t^* \in C^k([-1, 0], \mathbb{R})$.

(ii) If there exists $p > 0$ such that $x^*(t) = x^*(t + p)$, for all $t \in \mathbb{R}$ then $x_t^* \in C^\infty([-1, 0], \mathbb{R})$.

**[Extra credit] Ask questions:**
Feel free to enclose questions regarding the lecture and the exercises. That way it will be easier to address them during the exercise sessions. Please indicate whether your question should serve as a *basic question* or not.
Asking meaningful questions can earn you up to one extra point!