Homework Assignments

Dynamical Systems III-Delay Differential Equations

Isabelle Schneider, Alejandro López Nieto

http://dynamics.mi.fu-berlin.de/lectures/

due date: Wednesday, April 24, 2019, 12:00

Problem 1: Consider the scalar delay differential equation

$$\dot{x}(t) = x^2(t) - x(t-1).$$

The method of steps seen in the lecture ensures local existence of solutions given an initial condition $\phi \in C$. Let $t_{max} : C \to \mathbb{R}^+ \cup \{+\infty\}$ denote the function associating the maximal time of existence to an initial condition.

Prove or disprove:

(i) There exist initial conditions $\phi \in C$ such that $t_{max}(\phi) = +\infty$.

(ii) All solutions are globally defined, i.e. the maximal time of existence of solutions is $t_{max} \equiv +\infty$.

(iii) There exists a lower bound for the maximal time of existence, i.e. there exists $\varepsilon > 0$ such that $t_{max}(\phi) > \varepsilon$ for every $\phi \in C$.

(iv) Fix $t^* > 0$ and consider the subset of continuous functions

$$M(t^*) := \{\phi \in C \mid t_{max}(\phi) > t^*\} \subset C.$$

If we choose two different initial conditions $x_0, y_0 \in M(t^*)$, $x_0 \neq y_0$. Then $x_t \neq y_t$ for any $0 \leq t \leq t^*$.

Problem 2: Consider the DDE

$$\begin{aligned} \dot{x}(t) &= [x(t)^2 + x(t-1)^2 + 1]\, y(t), \\ \dot{y}(t) &= [x(t)^2 + x(t-1)^2 + 1]\, (x(t) - x^3(t)). \end{aligned}$$

Find the equilibria of the system, prove that all solutions remain bounded in time and show that the only solutions that do not oscillate around an equilibrium indefinitely are those satisfying $y(0)^2/2 + x(0)^4/4 - x(0)^2/2 = 0$.

[Extra credit] Ask questions:
Feel free to enclose questions regarding the lecture and the exercises. That way it will be easier to address them during the exercise sessions. Please indicate whether your question should serve as a *basic question* or not.
Asking meaningful questions can earn you up to one extra point!