Homework Assignments

Dynamical Systems III-Delay Differential Equations

Isabelle Schneider, Alejandro López Nieto

http://dynamics.mi.fu-berlin.de/lectures/

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Problem 1: Consider the scalar linear delay differential equation

$$\dot{x}(t) = ax(t) + bx(t-1),\ a, b \in \mathbb{R},\ b > 0,$$

with characteristic equation $\Delta(\lambda) = 0$, $\lambda \in \mathbb{C}$.
Prove that:

(i) If $\Delta(\lambda) = 0$, then $\Delta(\overline{\lambda}) = 0$.

(ii) $\Delta(\lambda)$ always has one positive real root if $a + b > 0$.

(iii) If $\lambda = \mu + i\nu \in \mathbb{C}$ solves the characteristic equation, then

$$|\nu| \in \{0\} \cup \left(\bigcup_{k \geq 1} [(2k-1)\pi, 2k\pi] \right).$$

(iv) There are only finitely many $\lambda = \mu + i\nu \in \mathbb{C}$ such that $\Delta(\lambda) = 0$ and $\nu \in [\pi, 2\pi]$.

(v) If $\Delta(\mu + i\nu) = 0$ and $\mu = 0$, then

$$\frac{\mathrm{d}\mu}{\mathrm{d}b} > 0.$$

Problem 2: Consider the scalar linear delay differential equation

$$\dot{x}(t) = Lx_t := -\int_{-1}^{0} \sin(\pi\theta) x(t+\theta) \mathrm{d}\theta.$$

(i) Find the equilibria of the system.

(ii) Compute the characteristic equation $\Delta(\lambda) = 0$, $\lambda \in \mathbb{C}$.

(iii) Use the characteristic equation to conclude that the equilibria are unstable.

[Extra credit] Ask questions:
Feel free to enclose questions regarding the lecture and the exercises. That way it will be easier to address them during the exercise sessions. Please indicate whether your question should serve as a *basic question* or not.
Asking meaningful questions can earn you up to one extra point!