Homework Assignments

Bifurcations: Theory and Applications

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`http://dynamics.mi.fu-berlin.de/lectures/`

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Problem 37: For each of the following points, find an example of a vector field which is:

(i) Hamiltonian, but not reversible.

(ii) Reversible, but not Hamiltonian.

(iii) Both Hamiltonian and reversible.

(iv) Neither Hamiltonian nor reversible.

Problem 38: Consider an R-reversible vector field

$$\dot{x} = f(x) \in \mathbb{R}^N, \qquad f(Rx) = -Rf(x),$$

with R linear, $R^2 = \text{id}$, $\text{Fix}(R) := \{x \in \mathbb{R}^N \mid Rx = x\}$.

(i) Consider a solution $x(t)$ satisfying $x(0), x(\tau) \in \text{Fix}(R)$ for $x(0) \neq x(\tau)$. Prove that $x(t)$ is periodic and determine its period. What can you say about the minimal period?

(ii) Consider a nonstationary periodic orbit $\gamma \subset \mathbb{R}^N$ which intersects $\text{Fix}(R)$. Show that γ intersects $\text{Fix}(R)$ at exactly 2 points.

(iii) Consider a solution $x(t)$ satisfying $x(0) \in \text{Fix}(R)$, and with α-limit set $\{x_-\} \subset \text{Fix}(R)$, a singleton such that $x_- \neq x(0)$. Prove that $x(t)$ is a homoclinic solution.

Problem 39: Consider a R-reversible C^1-vector field

$$\dot{x} = f(x) \in \mathbb{R}^N, \qquad f(Rx) = -Rf(x), \qquad f(0) = 0, \qquad A := f'(0),$$

with R linear, $R^2 = \text{id}$, $\text{Fix}(R) := \{x \in \mathbb{R}^N \mid Rx = x\}$.

(i) Assume $\det(A) \neq 0$. Prove that the dimension N must be even.

(ii) Let N be odd and

$$\dim \text{Fix}(R) = (N+1)/2.$$

Assume $\dim \text{Ker} A < 2$. Show that there exists a local curve of equilibria in $\text{Fix}(R)$, near 0.

Problem 40: Consider a smooth vector field

$$(1)\ \dot{x} = f(\lambda, x), \qquad x \in \mathbb{R}^2, \qquad \lambda \in \mathbb{R}, \qquad f(\lambda, 0) \equiv 0, \qquad D_x f(\lambda, 0) = \begin{pmatrix} \lambda & -1 \\ 1 & \lambda \end{pmatrix},$$

together with the iteration

$$(2)\ x_{n+1} = \phi_T(\lambda, x_n),$$

where $\phi_T(\lambda, \cdot)$ is the time-T map of (1).

Characterize the parameter values λ, T for which the fixed point $x = 0$ in (2) undergoes subharmonic bifurcation. Discuss and interpret how the subharmonic bifurcation in (2) relates to the Hopf bifurcation from 0 for the ODE (1) at $\lambda = 0$.