Homework Assignments

Bifurcations: Theory and Applications

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Problem 41: Consider the periodically forced smooth ODE on $\mathbb{R}^N$

$$\dot{x} = f(\lambda, t, x),$$

where $\lambda \in \mathbb{R}^2$, $f(\lambda, t+1, x) = f(\lambda, t, x)$, $f(\lambda, t, 0) \equiv 0$.
Give sufficient conditions on f under which subharmonic solutions with minimal period $m \in \mathbb{N}$ bifurcate from the trivial equilibrium $x = 0$. Prove your claims.

Problem 42: Consider the C^2-ODE on $\mathbb{R}^2$

$$\begin{aligned} \dot{x} &= f(x, y), \\ \dot{y} &= -f(y, x). \end{aligned}$$

Assume $\partial_2 f(0,0) \neq 0$, and $f(x,y) = f(-x,y) = -f(x,-y)$ for all x, y. Show that $x = y = 0$ is an equilibrium. Also show that there exists a family of periodic orbits near 0. Discuss their spatio-temporal symmetry.

Problem 43: Consider the representation of $D_3 \times S^1$ on $\mathbb{C}^2$ given by

$$\begin{aligned} \varphi(z_1, z_2) &= (\mathrm{e}^{2\pi \mathrm{i}/3} z_1, \mathrm{e}^{-2\pi \mathrm{i}/3} z_2), \\ \sigma(z_1, z_2) &= (z_2, z_1), \\ \vartheta(z_1, z_2) &= (\mathrm{e}^{\mathrm{i}\vartheta} z_1, \mathrm{e}^{\mathrm{i}\vartheta} z_2). \end{aligned}$$

Where D_3, generated by φ, σ is the symmetry group of the equilateral triangle. Determine the isotropy subgroups and their fixed-point spaces.

Problem 44: Consider a ring of coupled Stuart-Landau oscillators

$$\dot{z}_i = (\lambda + i + \gamma |z|_i^2) z_i + a(z_{i-1} - 2z_i + z_{i+1}), \qquad i \bmod 3, \qquad \gamma, z_i \in \mathbb{C}, \qquad \lambda, a \in \mathbb{R}.$$

Find the values λ at which 0 undergoes Hopf bifurcation and discuss the symmetry of the bifurcating solutions.