Homework Assignments

Bifurcations: Theory and Applications

Bernold Fiedler, Alejandro López Nieto

http://dynamics.mi.fu-berlin.de/lectures/

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Problem 21: In the setting of the Arnol'd-Takens-Bogdanov bifurcation from the previous exercise sheet, consider the modification

$$\begin{aligned}\dot{x}_1 &= x_2,\\ \dot{x}_2 &= \lambda_1 x_1 + \lambda_2 x_2 + x_1^2 + x_1 x_2.\end{aligned}$$

Determine the parameters (λ_1, λ_2) near zero where:

(i) Stationary bifurcations could occur.

(ii) Hopf bifurcation could occur.

Discuss what happens in the phase planes, locally near $x = 0$. Just try, as far as you get.

Problem 22: Consider the real ordinary differential equation

$$\dot{x} = \lambda_1 + \lambda_2 x + x^3.$$

The set of equilibria determines a surface, which can be parametrized over (λ_2, x), instead of solving for x as a function of (λ_1, λ_2). The number of equilibria x, near any given $\lambda = (\lambda_1, \lambda_2)$, can only change where the implicit function theorem for $x = x(\lambda)$ fails. Determine the number of equilibria and their stability, depending on the values of the real parameters λ_1 and λ_2. Sketch the global phase portraits for each relevant region in the parameter plane.

Problem 23: Consider the unfolding of the van der Pol oscillator

$$\begin{aligned}\varepsilon\dot{x} &= y + x - \tfrac{1}{3}x^3,\\ \dot{y} &= \lambda - x.\end{aligned}$$

Use center manifold theory and time rescalings to discuss the dynamics of the system for fixed λ and $\varepsilon \searrow 0$. Sketch the phase portraits for $\lambda > 1$ and $\lambda < 1$.
Do nonstationary periodic orbits exist?

Problem 24: In the setting of problem 23, discuss the bifurcation of the equilibrium $(x, y) = (\lambda, \frac{1}{3}\lambda^3 - \lambda)$ for λ crossing 1 and fixed ε.
Which solutions bifurcate? How do you reconcile this effect with the observations of problem 23, for λ close to 1 and small $\varepsilon > 0$?