

Homework Assignments

Bifurcations: Theory and Applications

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due date: **Wednesday, December 18, 2019, 12:00**

Let ρ be a representation of the group Γ on a vector space X , and K a subgroup of Γ . For $x \in X$ consider Γ_x , the isotropy of x , and X_K , the space of K -fixed vectors.

Problem 29: For any $\gamma \in \Gamma$ show

- (i) $\Gamma_{\gamma x} = \gamma \Gamma_x \gamma^{-1}$.
- (ii) $X_{\gamma K \gamma^{-1}} = \gamma X_K$.
- (iii) Γ_x is a normal subgroup of Γ if, and only if, the isotropy of any y in the Γ -orbit of x is the same.

Problem 30: Assume that K is a normal subgroup of Γ . Show that ρ defines a representation of the factor group Γ/K on X_K .

Problem 31: Consider the complex shift representation ρ of the group $\Gamma = (\mathbb{R}, +)$ on $X = L^2(\mathbb{R}, \mathbb{C})$.

- (i) Determine all finite-dimensional irreducible subrepresentations of ρ .
- (ii) Show that X possesses closed \mathbb{R} -invariant subspaces, and hence it is not irreducible.
- (iii) Does ρ possess any infinite-dimensional, closed, irreducible subspace?

Problem 32: Consider the Laplacian $\Delta := \partial_{x_1}^2 + \partial_{x_2}^2 + \partial_{x_3}^2 : H_\ell \rightarrow H_{\ell-2}$, where H_ℓ is the space of real valued homogeneous polynomials in three variables. Compute $V_\ell := \text{Ker} \Delta|_{H_\ell}$ and determine $\dim V_\ell$.

Hint: Note that the elements of H_ℓ are of the form $\sum_{k=0}^{\ell} x_3^k a_k(x_1, x_2)$, where a_k are homogeneous polynomials of degree $\ell - k$, in two variables!